Tyler started to wish he had paid more attention in Algebra.

Straight Line Graphs

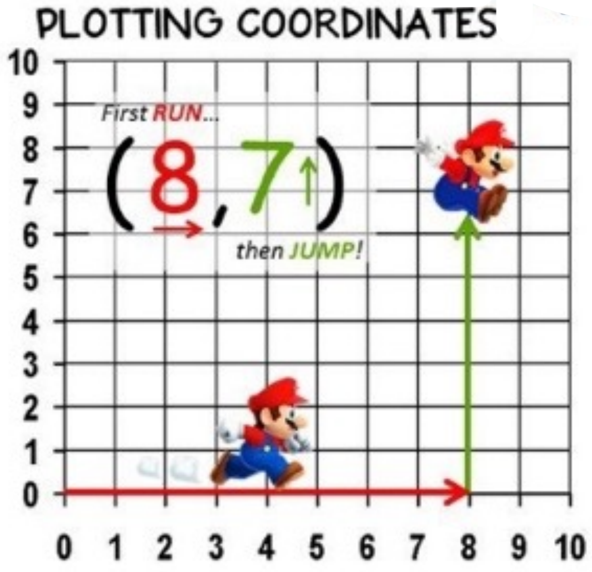

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Table Of Contents

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What is The X and Y Axis

What is the x axis and what is the y axis?

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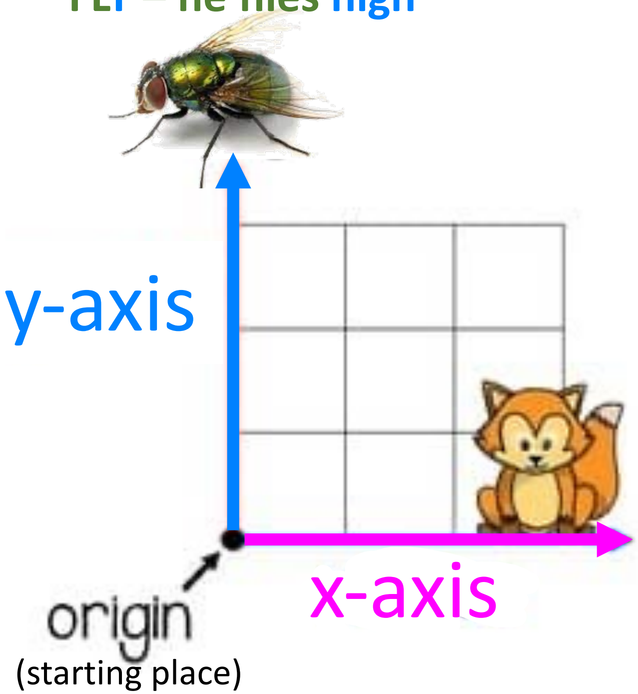
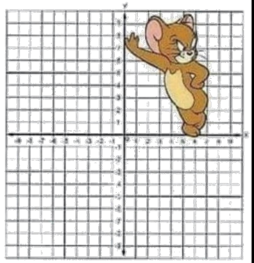
FLY – he flies high

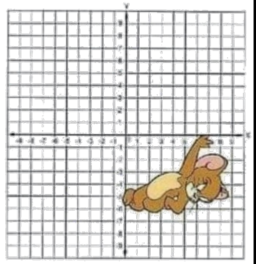


Sly FOX – keeps it on the down low and walks

JERRY



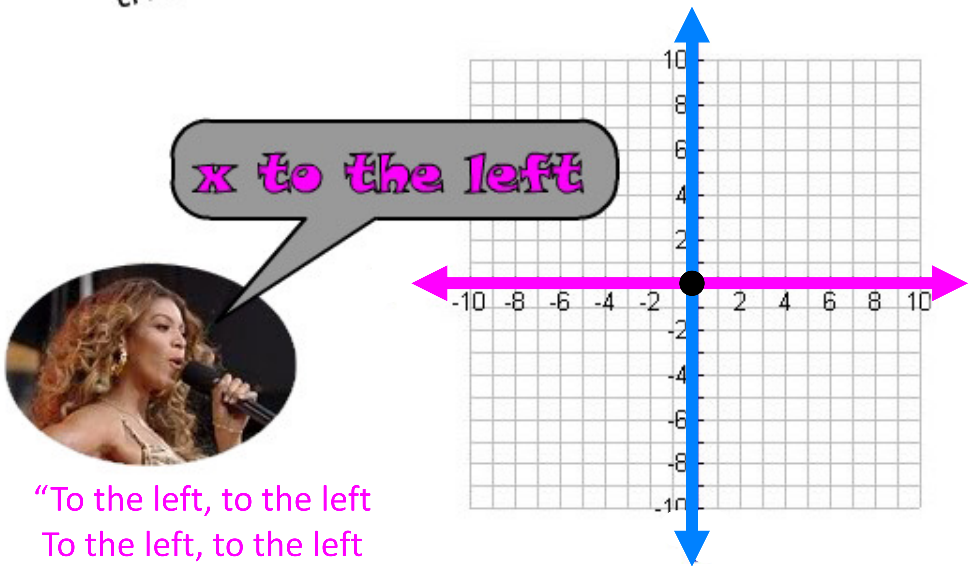
JERRX



On the x axis count left or right

On the y axis count up or down

The x-axis and y-axis trick:



"To the left, to the left
To the left, to the left
Mm
You must not know 'bout me
You must not know 'bout me
I can have another you by tomorrow
So don't you ever for a second get to thinking
You're irreplaceable"

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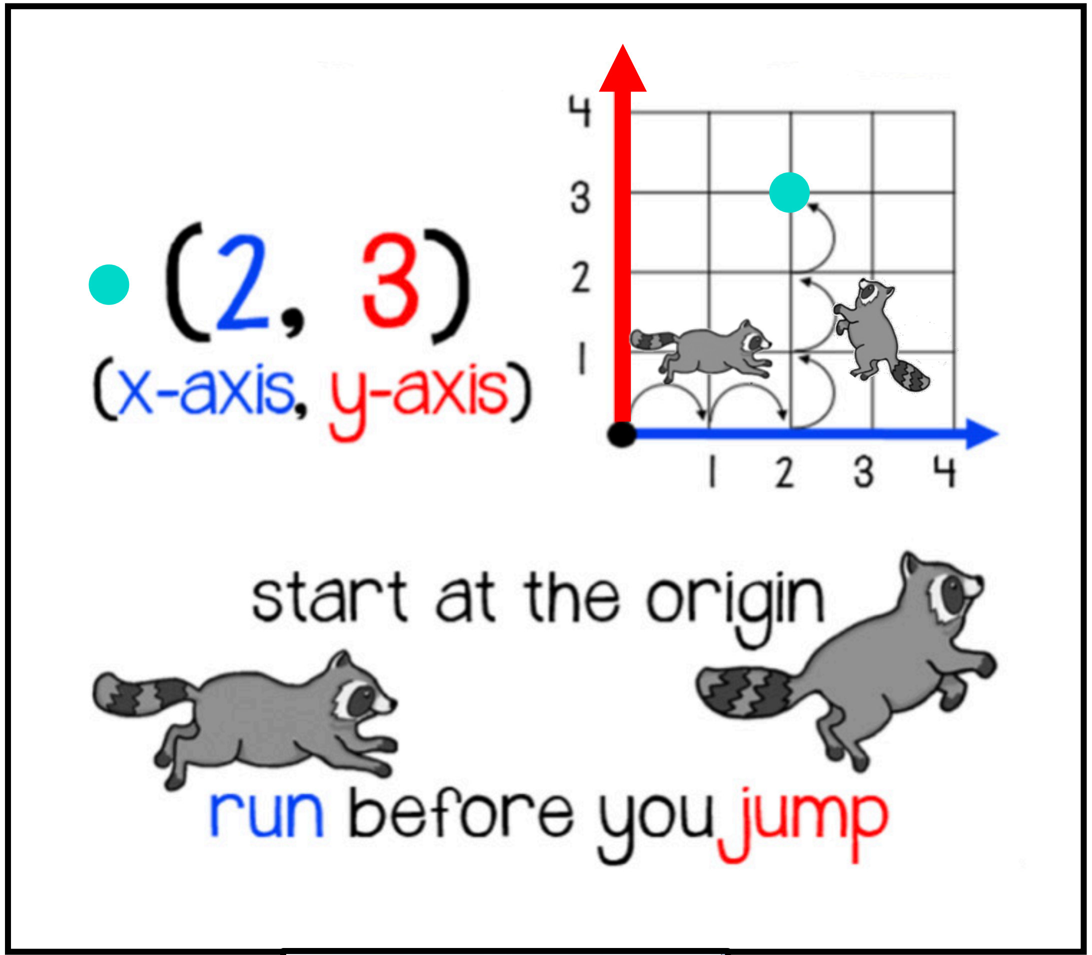
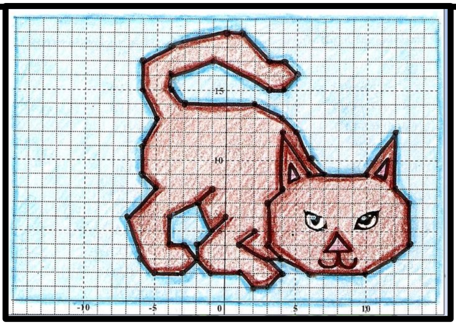
What Are Coordinates?

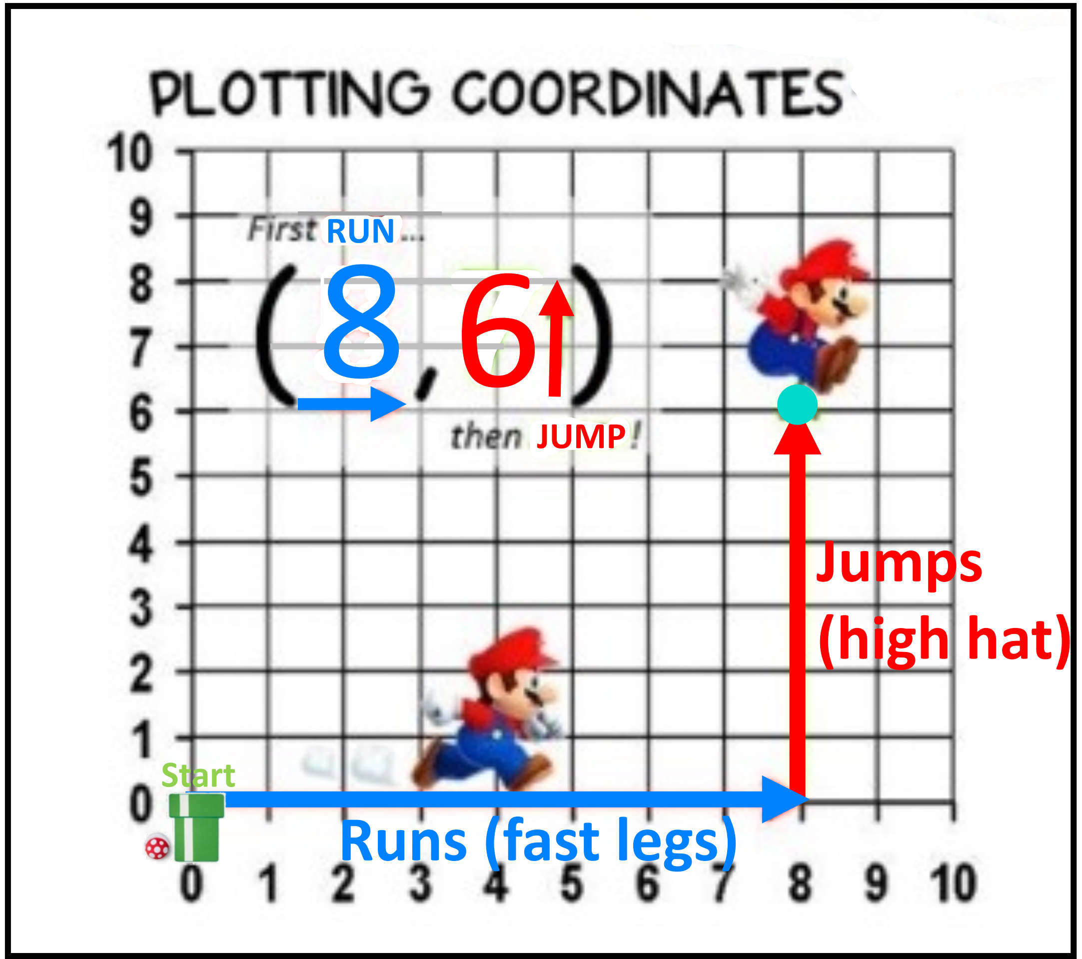
Animal Style Coordinates

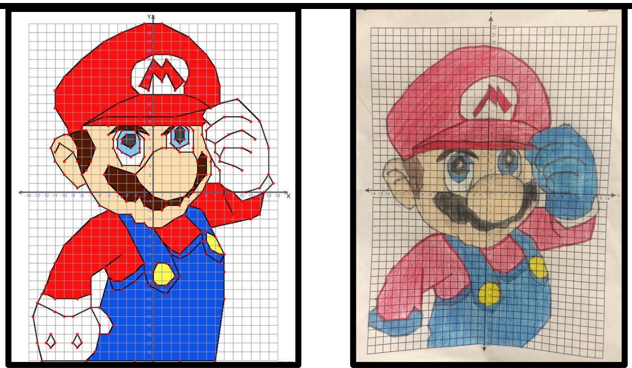




"I must run first before I pounce and jump on my target"

Mario Style Coordinates





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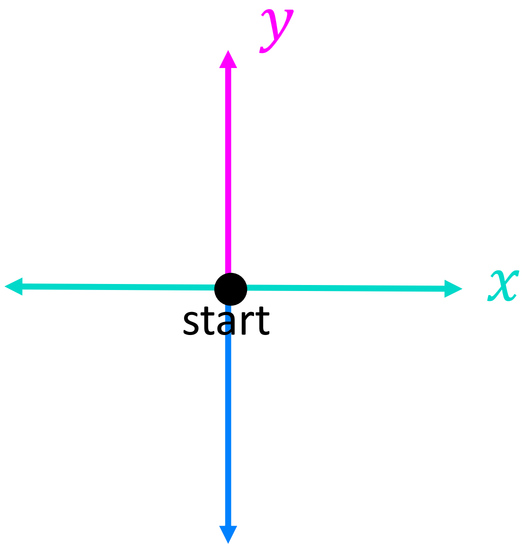
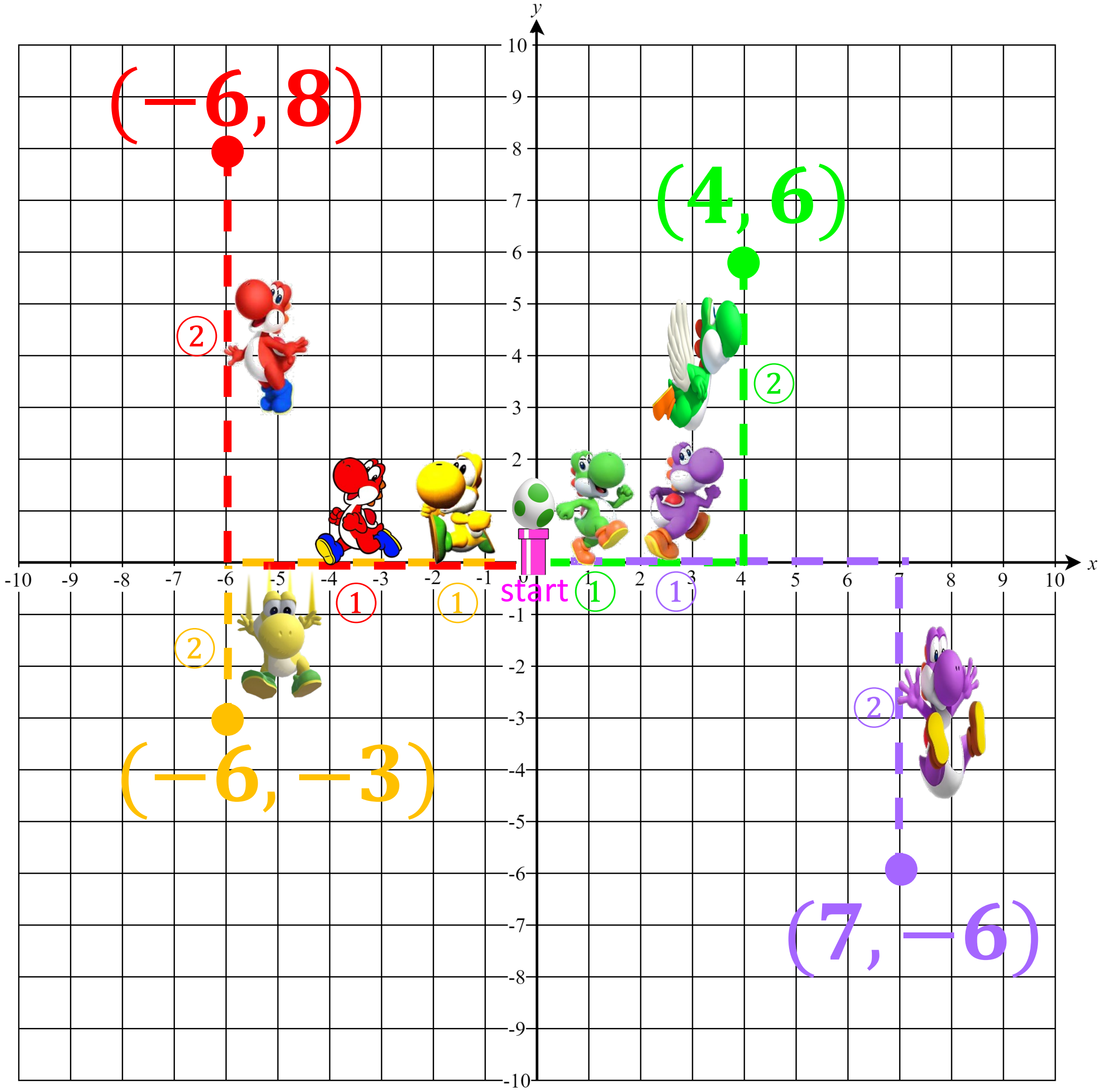
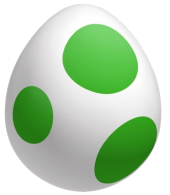
Yoshi Style Coordinates

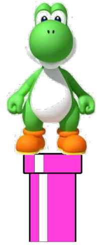


(Run , Jump) or **(Run, Drop)**

(x, y) (x, y)

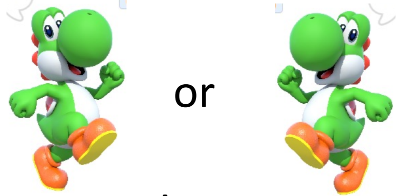






Ready To Start

1

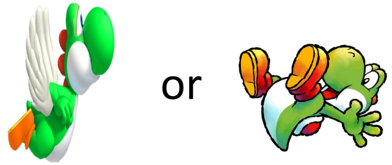


or

run left or right

then..

2



or

jump or drop

and the process starts again

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What Does Gradient/Slope Mean?

Slope Dude

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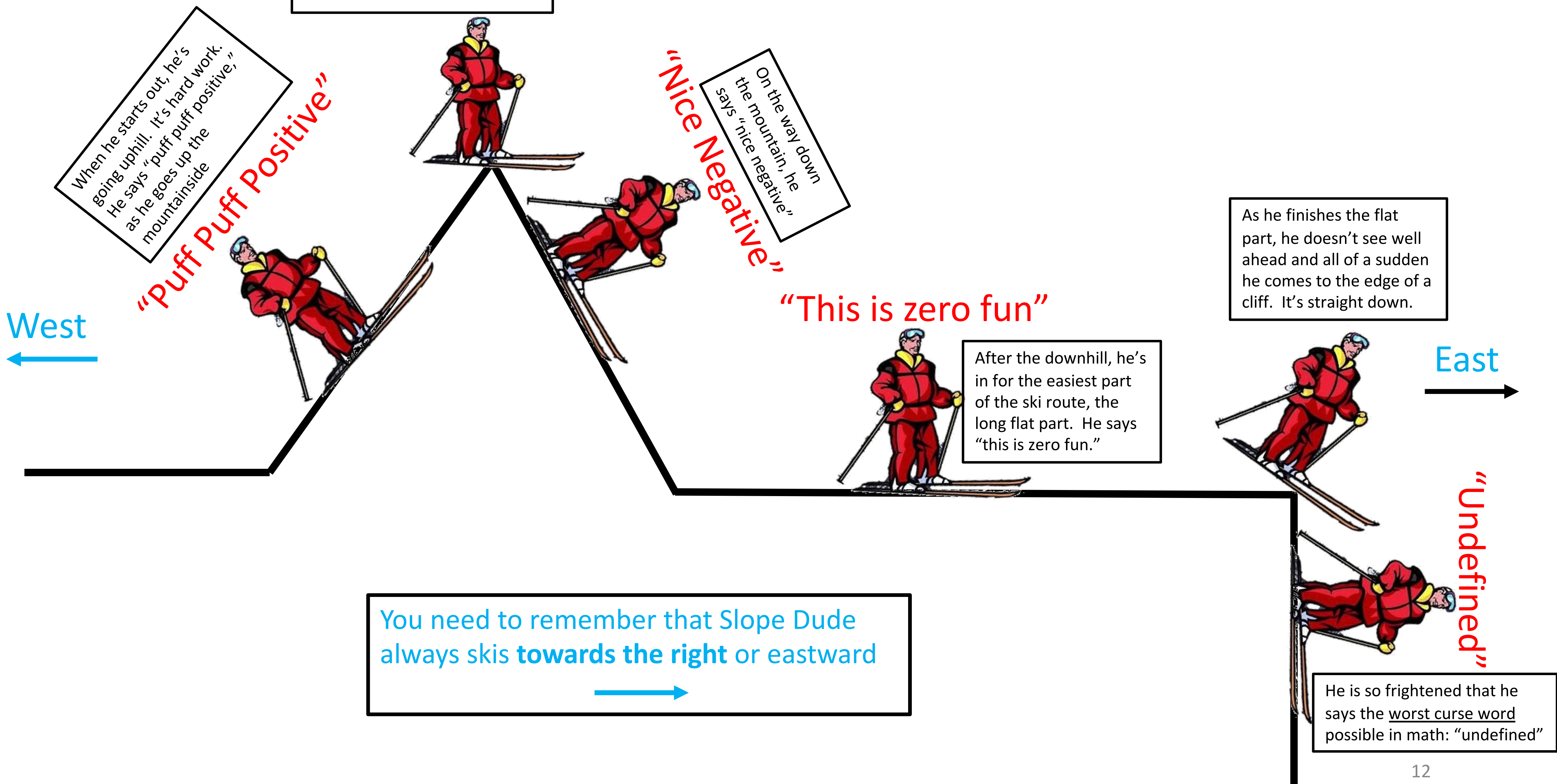

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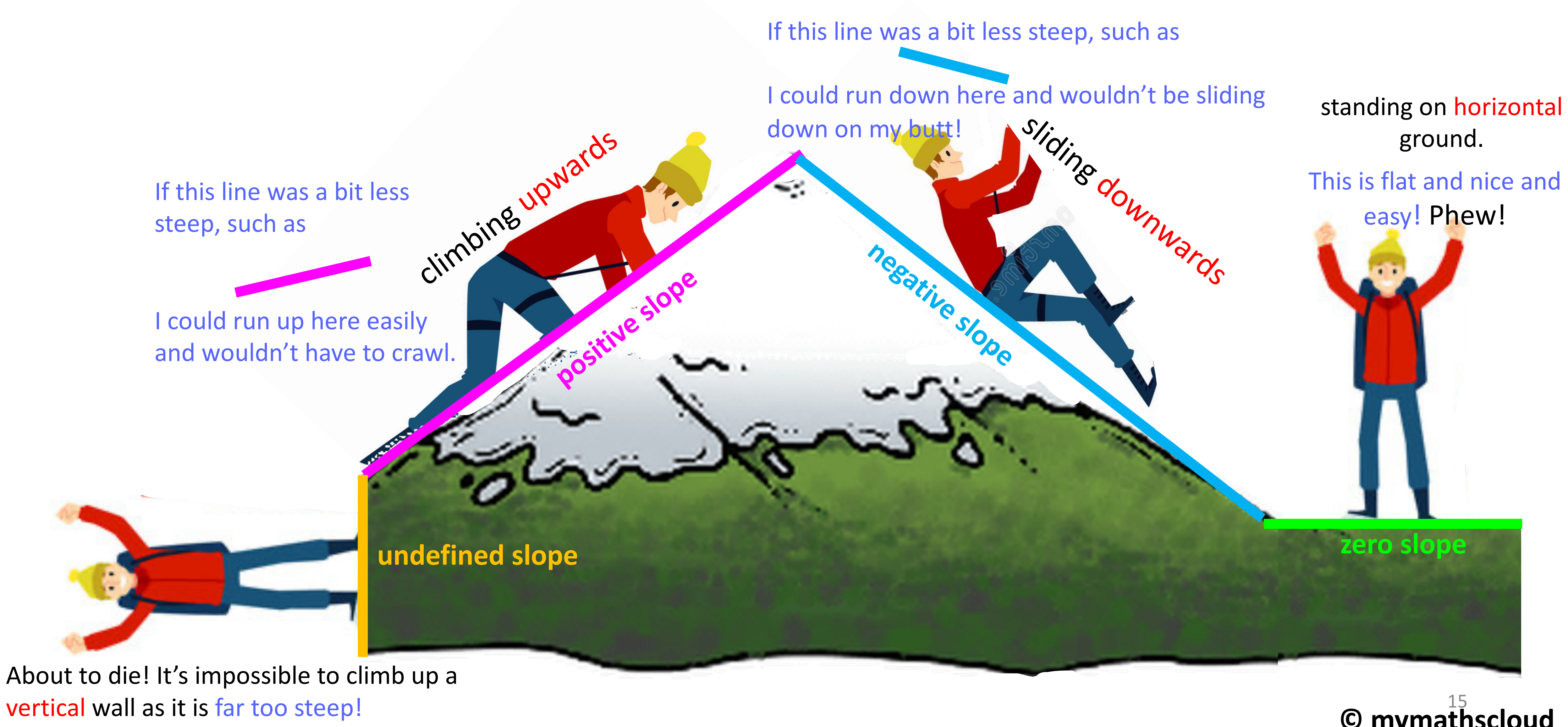
Gradient/Slope Explained In More Detail

The slope/gradient is measure of how steep a line is
The slope/gradient also tells us about the direction of a line



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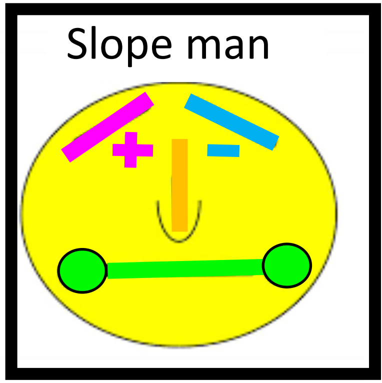
Understanding slope means understanding two things: steepness and direction

Let's use a skater to demonstrate this.



The slope of the line on the left above is **steeper** than the slope of the line on the right.
In addition, the skaters are going down the ramp from the left to the right. This means the slope **decreasing,** or negative.

How about if the skaters were going up the ramp? This would mean that the slope is **Increasing,** or positive.

So, slope measures the **direction** of the line – whether or not the skater is going up the ramp (positive slope) or going down the ramp (negative slope). It also measures the **steepness** of a line - the steeper the ramp the larger the value will be for the slope.

A skater doesn't always skate on an incline though. A skater could also skate on flat ground, which would mean that there is no steepness to the line and therefore it would be defined as **zero slope.** Or what about if the skater was a show-off and wanted to go straight down the side of a building or ramp? This is known as an **undefined slope**.

Four Different Types of Slopes for Directions

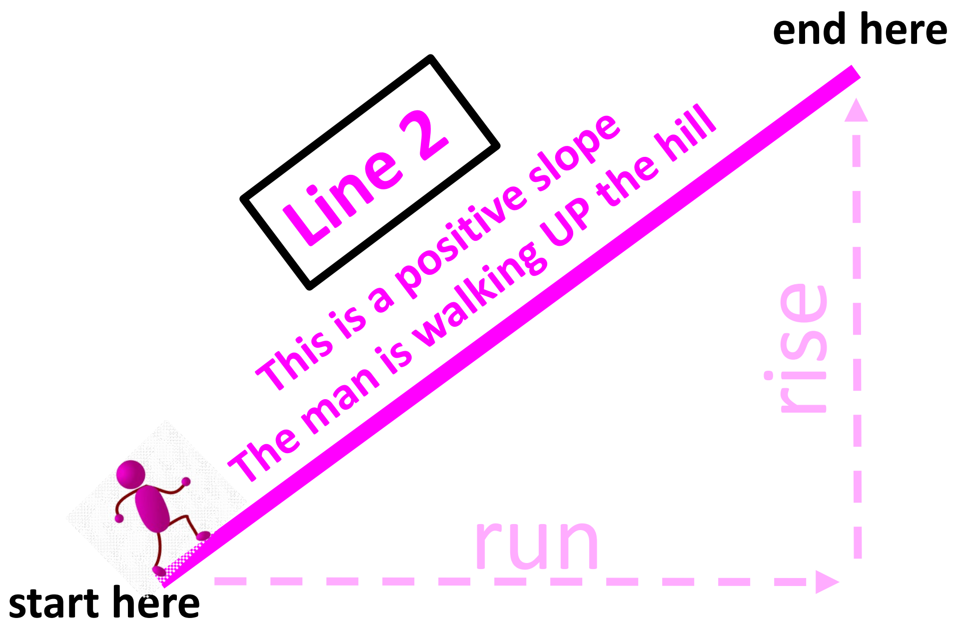
Positive (increasing)	Negative (decreasing)	Zero (horizontal line)	Undefined (vertical line)

Examples of Slopes for Steepness

Not steep Slope=0.1	A little steeper Slope=1	Even steeper Slope=2	Very steep Slope=4
Not steep Slope=−0.1	Not steep Slope=−1	Not steep Slope=−2	Not steep Slope=−4

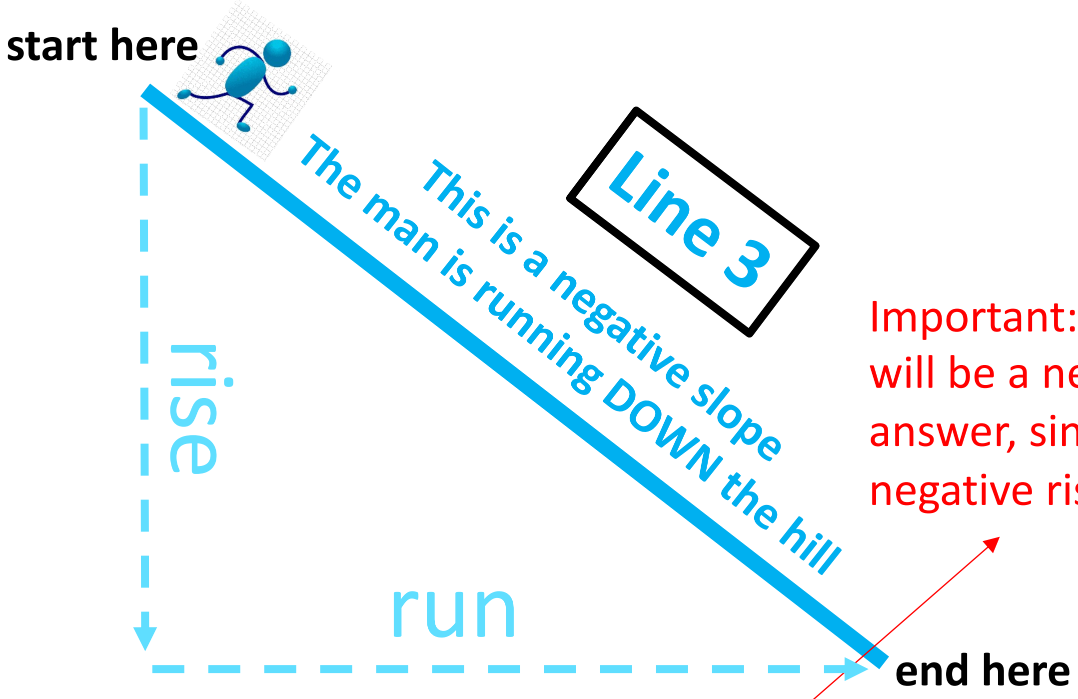
We will see how to find the numbers for the slope over the next few pages

Let's look at our four different types of lines in a bit more detail



$$\text{slope} = \frac{\text{how much} \uparrow}{\text{how much} \rightarrow} = \frac{rise}{run}$$

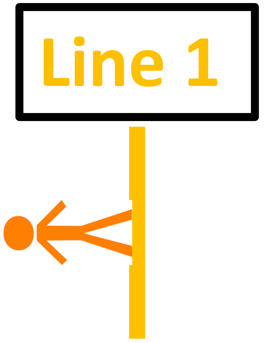
(the slope is positive since it increases from LEFT to RIGHT)



Important: This rise will be a negative answer, since it is a negative rise i.e. a fall)

$$\text{slope} = \frac{\text{how much} \downarrow}{\text{how much} \rightarrow} = \frac{rise}{run}$$

(the slope is negative since it decreases from LEFT to RIGHT)



the slope is undefined (the man can't walk up that line)

the slope is zero (the man is walking on flat ground)

Note: $\frac{\text{rise}}{\text{run}}$ is just the same as $\frac{\text{change in } y}{\text{change in } x}$

We will see on the following pages how to get the actual value of the slope

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How Do We Calculate The Gradient/Slope?

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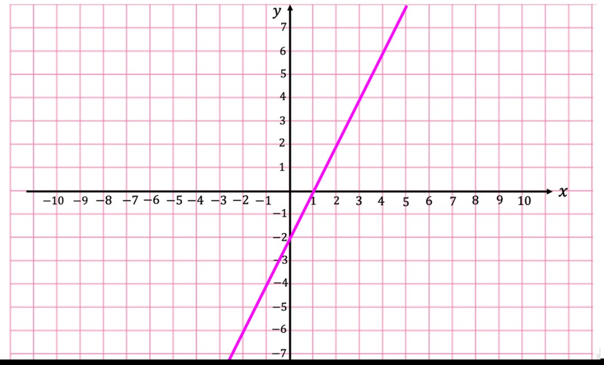
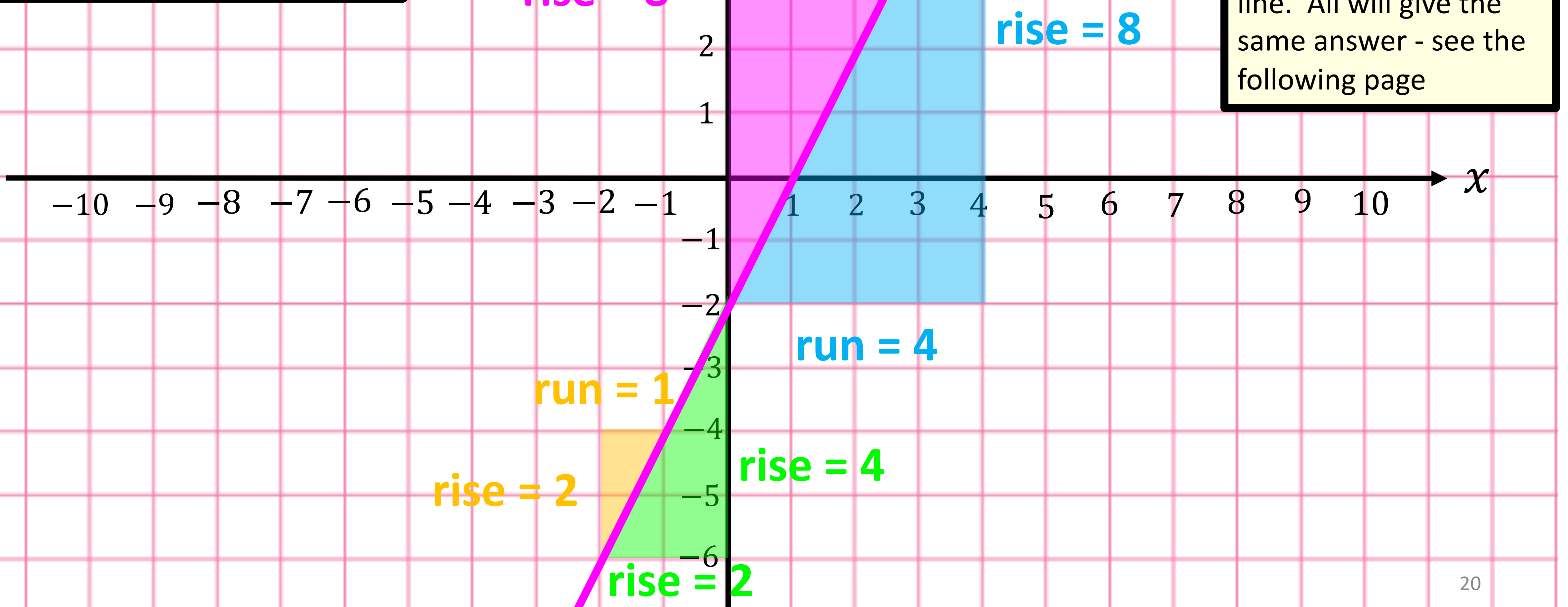
Way 1:
From A Graph-
Build A Triangle

Example 1: Consider the following pink line





Method:
We can build ANY of the triangles shown (it doesn't matter what size they are or whether they are above or below the line) for the given pink line. All will give the same answer - see the following page

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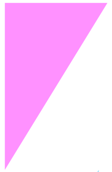
Calculating the value of the slope/gradient

As mentioned, we can build any triangle to find the slope/gradient. It doesn't matter where we form it above or below the line. We use the letter m to represent slope. Carrying on from example 1 above:

The formula for slope is $\text{slope} = m = \frac{rise}{run}$

Using the pink triangle  : $\mathbf{m} = \frac{rise}{run} = \frac{\mathbf{8}}{\mathbf{4}} = \mathbf{2}$

Using the blue triangle. : $\mathbf{m} = \frac{rise}{run} = \frac{\mathbf{8}}{\mathbf{4}} = \mathbf{2}$

Using the orange triangle. : $\mathbf{m} = \frac{rise}{run} = \frac{\mathbf{2}}{\mathbf{1}} = \mathbf{2}$

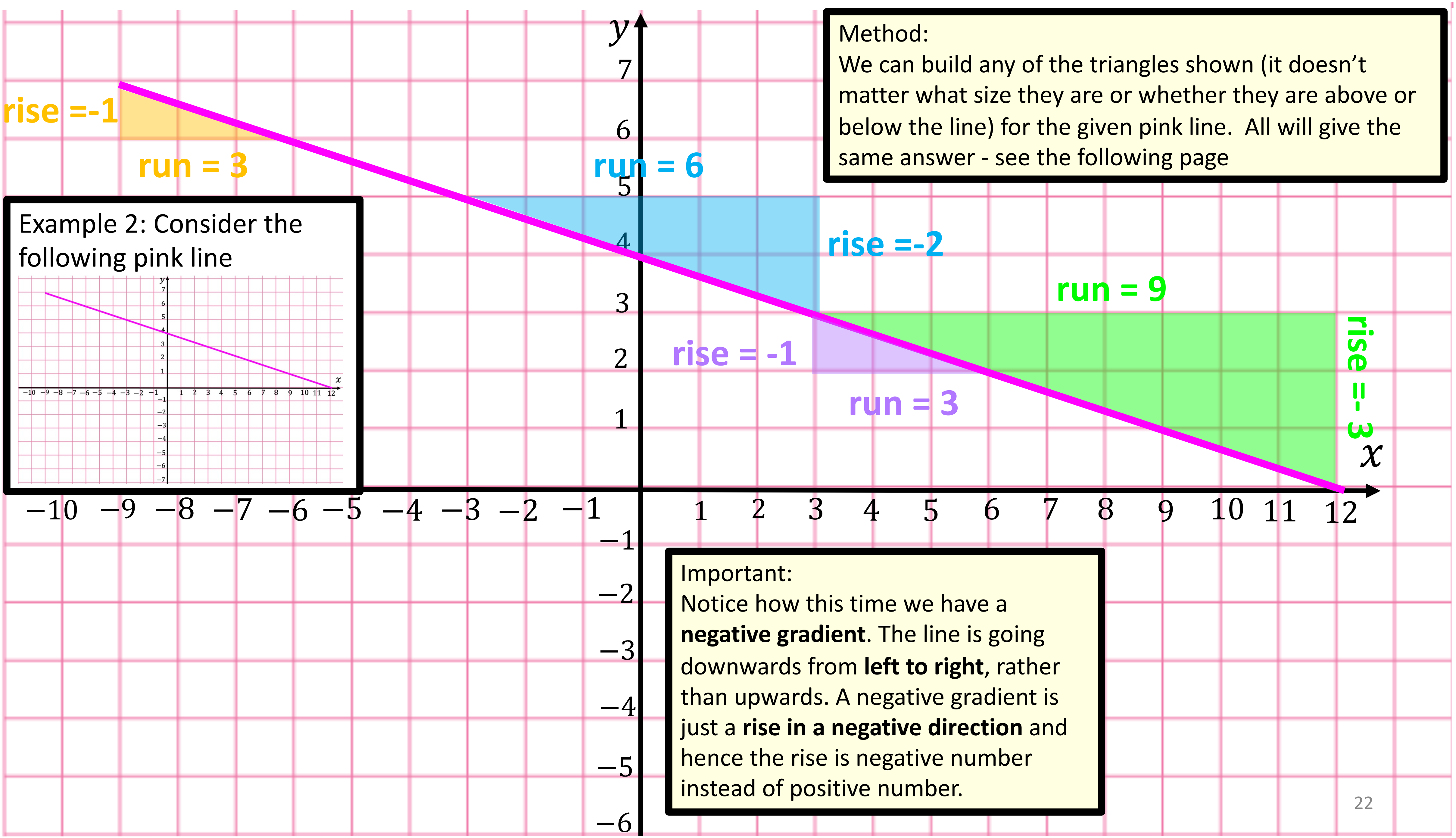
Using the green triangle. : $\mathbf{m} = \frac{rise}{run} = \frac{\mathbf{4}}{\mathbf{2}} = \mathbf{2}$

Notice how all give the same answer for the slope which is 2. Some just need to be simplified in order to see that they give the same value!

$$\text{slope} = m = 2$$

Method:
We can build any of the triangles shown (it doesn't matter what size they are or whether they are above or below the line) for the given pink line. All will give the same answer - see the following page

Example 2: Consider the following pink line



Important:
Notice how this time we have a **negative gradient**. The line is going downwards from **left to right**, rather than upwards. A negative gradient is just a **rise in a negative direction** and hence the rise is negative number instead of positive number.

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Calculating the value of the slope/gradient

$$\text{slope} = m = \frac{rise}{run}$$

Note: our rise is negative since we fall this time (negative rise)

Using the blue triangle $: m = \frac{rise}{run} = \frac{-2}{6} = -\frac{1}{3}$

Using the orange triangle $: m = \frac{rise}{run} = \frac{-1}{3}$

Using the green triangle $: m = \frac{rise}{run} = \frac{-3}{9} = -\frac{1}{3}$

Using the purple triangle $: m = \frac{rise}{run} = \frac{-1}{3}$

$$\text{slope} = m = -\frac{1}{3}$$

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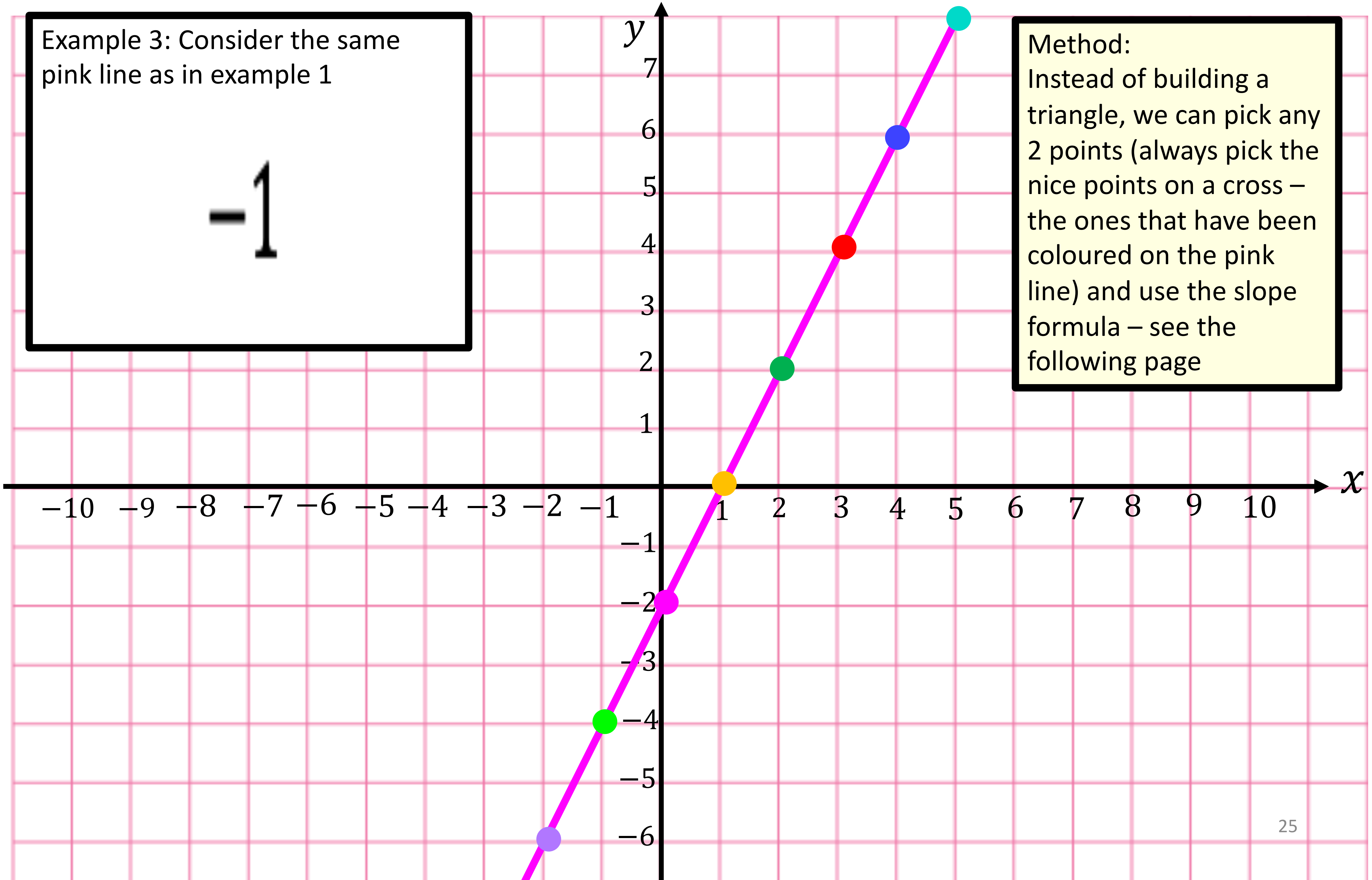
Way 2 :
From A Graph -
Pick <u>Any</u> Two Points
On A Line

Example 3: Consider the same pink line as in example 1

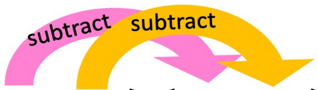
-1

Method:
Instead of building a triangle, we can pick any 2 points (always pick the nice points on a cross – the ones that have been coloured on the pink line) and use the slope formula – see the following page



Let's first learn what the formula for the slope is when we have two points. Let's generally call the points (x_1, y_1) and (x_2, y_2)

Way 1 (left to right)

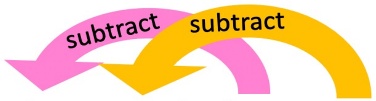


$(x_1, y_1), (x_2, y_2)$

Formula

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Way 2 (right to left)



$(x_1, y_1), (x_2, y_2)$

Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Method:

This formula basically says:
we subtract the y coordinates and divide by the answer we get by subtracting the x coordinates

$$\frac{y_2 - y_1}{x_2 - x_1} \; or \; \frac{y_2 - y_1}{x_1 - x_2}$$

The formula should make sense because

$\frac{\text{rise}}{\text{run}} = \frac{\updownarrow}{\leftrightarrow}$ which is just $\frac{\text{change in y}}{\text{change in x}}$

Why are there 2 ways? It doesn't matter which way round we subtract, as long as we stay consistent!

So, for our graph for example 3 on the previous page, we had the following coordinates

● $(5,8)$ ● $(4,6)$ ● $(3,4)$ ● $(2,2)$ ● $(1,0)$ ● $(0,-2)$ ● $(-1,-4)$ ● $(-2,-6)$

Pick ANY pair of coordinates. Let's choose $(5,8)$ and $(0,-2)$

Way 1

$$m = \frac{8 - -2}{5 - 0} = \frac{8+2}{5} = 2$$

Way 2

$$m = \frac{-2 - 8}{0 - 5} = \frac{-10}{-5} = 2$$

slope = $m = 2$

Note: picking any two coordinates would have still given us the same answer

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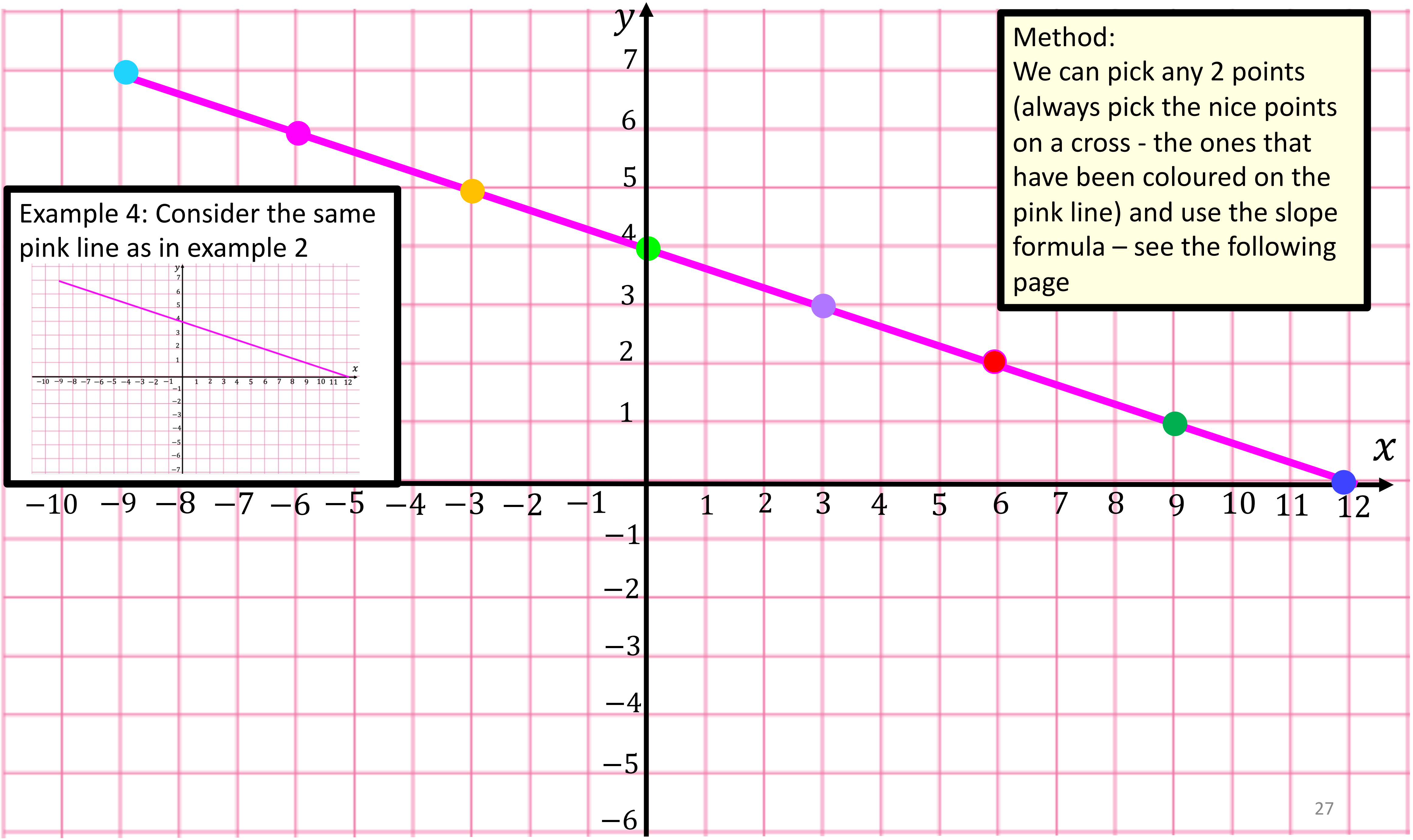
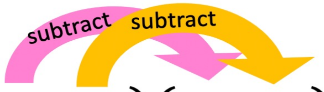
Example 4: Consider the same pink line as in example 2

Method:
We can pick any 2 points (always pick the nice points on a cross - the ones that have been coloured on the pink line) and use the slope formula – see the following page



Recall the slope formula:

Way 1 (left to right)

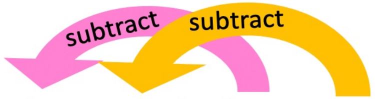


$$(x_1, y_1), (x_2, y_2)$$

Formula

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Way 2 (right to left)



$$(x_1, y_1), (x_2, y_2)$$

Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

So, for our graph for example 4 on the previous page, we had the coordinates

● $(-9,7)$ ● $(-6,6)$ ● $(-3,5)$ ● $(0,4)$ ● $(3,3)$ ● $(6,2)$ ● $(9,1)$ ● $(12,0)$

Pick ANY pair of coordinates. Let's choose $(-3,5)$ and $(3,3)$

Way 1

$$m = \frac{5-3}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$$

Way 2

$$m = \frac{3-5}{3--3} = \frac{-2}{6} = -\frac{1}{3}$$

$$\text{slope} = m = -\frac{1}{3}$$

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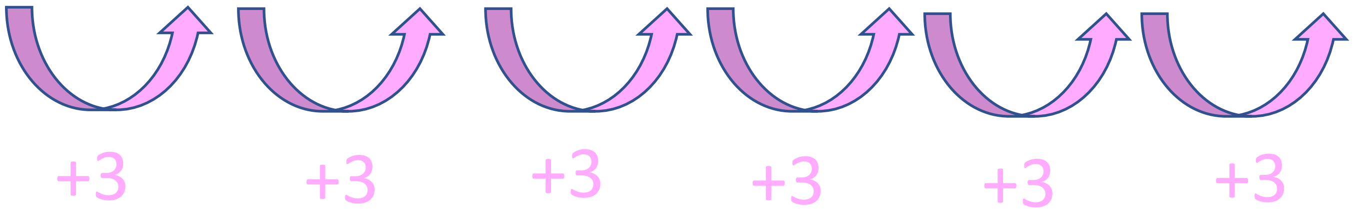
Way 3 : From A Table Of Values

When given a table of values, we could just plot all the points, form the line and then use the previous methods learnt such as building triangles or picking two points on the line, but we don't have to! There is a quicker way when we're given table of values!

x	-3	-2	-1	0	1	2	3
y	-11	-8	-5	-2	1	4	7



The slope is just the constant number that y is changing by. Here we keep adding 3, so the slope is 3

$$\text{slope} = m = 3$$

Note: This only works because the x values are changing by one each time in the table. If the table only consisted of even values for x say -2, 0, 2 4 or only odd values say $-3, -1, 1, 3$ then we would get twice the slope.

Sometimes we'll be a given a table and sometimes we'll need to build it. We will see how to build a table later on in the how to graph a line section.

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Way 4 : From Two Coordinates

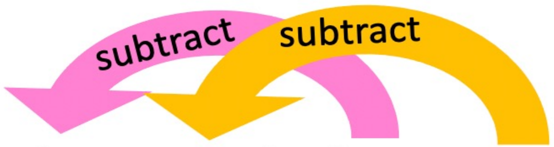
We have already seen how to do this when we dealt with choosing two coordinates on the line! Here we are just already given the coordinates for free and don't have to pick them off the graph!

Way 1

$$(x_1, y_1)(x_2, y_2)$$

$$\text{slope} = m = \frac{y_1 - y_2}{x_1 - x_2}$$

Way 2



$$(x_1, y_1)(x_2, y_2)$$

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Remember, it doesn't matter which way round we subtract, as long as we stay consistent (hence we have 2 ways)

For example: Find slope of the line passing through the points $(-1,2)$ and $(4,-5)$

$$\frac{2--5}{-1-4} = \frac{7}{-5} = -\frac{7}{5} \quad \text{or} \quad \frac{-5-2}{4--1} = \frac{-7}{5} = -\frac{7}{5}$$

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Way 5 :
From The Equation Of A Line

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The equation of a line looks like $y = mx + c$

There are 2 values that are important: m and c. We have already seen that m represents the slope

$$y = mx + c$$

The gradient/slope is just this value of m right here in front of x

Note: We will see what the c value means in a bit

Let's look at some examples

$y = x - 2$	$y = 2x - 1$	$y = -x + 4$	$y = -2 + 3x$	$y = 2 - 4x$	$x = 4$	$y = 5$
$y = x + 2$ means $y = 1x + 2$ gradient $= 1$	gradient $= 2$	$y = x + 2$ means $y = -1x + 4$ gradient $= -1$	Need to re-order this first $y = 3x - 2$ gradient $= 3$	Need to re-order this first $y = -4x + 2$ gradient $= -4$	This is a vertical line since x is the same value the whole time. The gradient here is undefined	This is a horizontal line since y is the same value the whole time. $y = 5$ is like writing $y = 0x + 5$ gradient $= 0$

$y + x = 4$	$y - 2x = 5$	$2x + 4y = 5$	$5x - 2y = 7$	$2x + 3y - 1 = 0$	$x + 2y + 5 = 0$
We need to use algebra to re-arrange to make y the subject $y = -x + 4$ gradient $= -1$	We need to use algebra to re-arrange to make y the subject $y = 2x + 5$ gradient $= 2$	We need to use algebra to re-arrange to make y the subject $4y = -2x + 5$ $y = \frac{-2x + 5}{4}$ $y = -\frac{1}{2} + \frac{5}{4}$ gradient $= -\frac{1}{2}$	We need to use algebra to re-arrange to make y the subject $-2y = -5x + 7$ $y = \frac{-5x + 7}{-2}$ $y = \frac{5}{2}x - \frac{7}{2}$ gradient $= \frac{5}{2}$	We need to use algebra to re-arrange to make y the subject $3y = -2x + 1$ $y = \frac{-2x + 1}{3}$ $y = -\frac{2}{3}x + \frac{1}{3}$ gradient $= -\frac{2}{3}$	We need to use algebra to re-arrange to make y the subject $2y = -x - 5$ $y = \frac{-x - 5}{2}$ $y = -\frac{1}{2}x - \frac{5}{2}$ gradient $= -\frac{1}{2}$

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What Is The Y Intercept And How Do We Find It?

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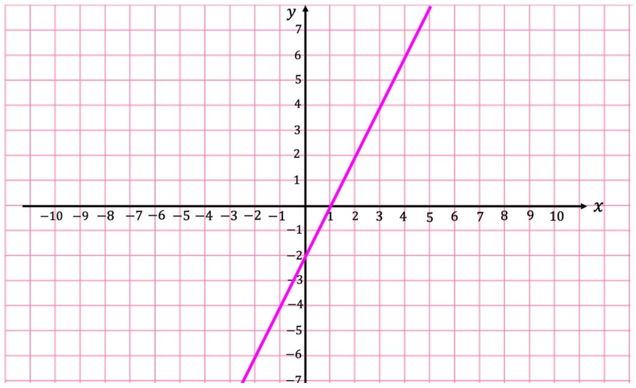
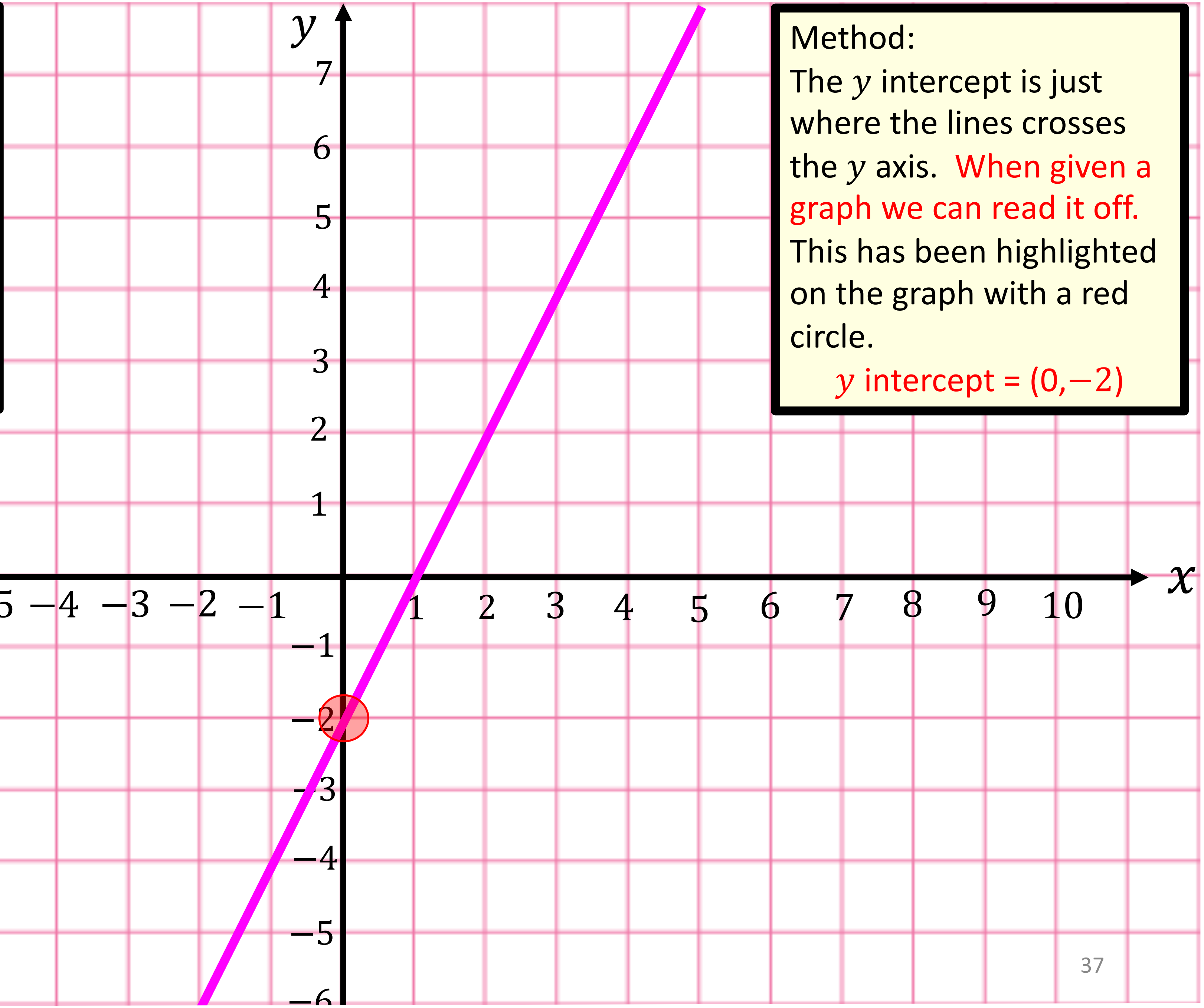
Way 1:
From A Graph

Example 5: Consider the following pink line

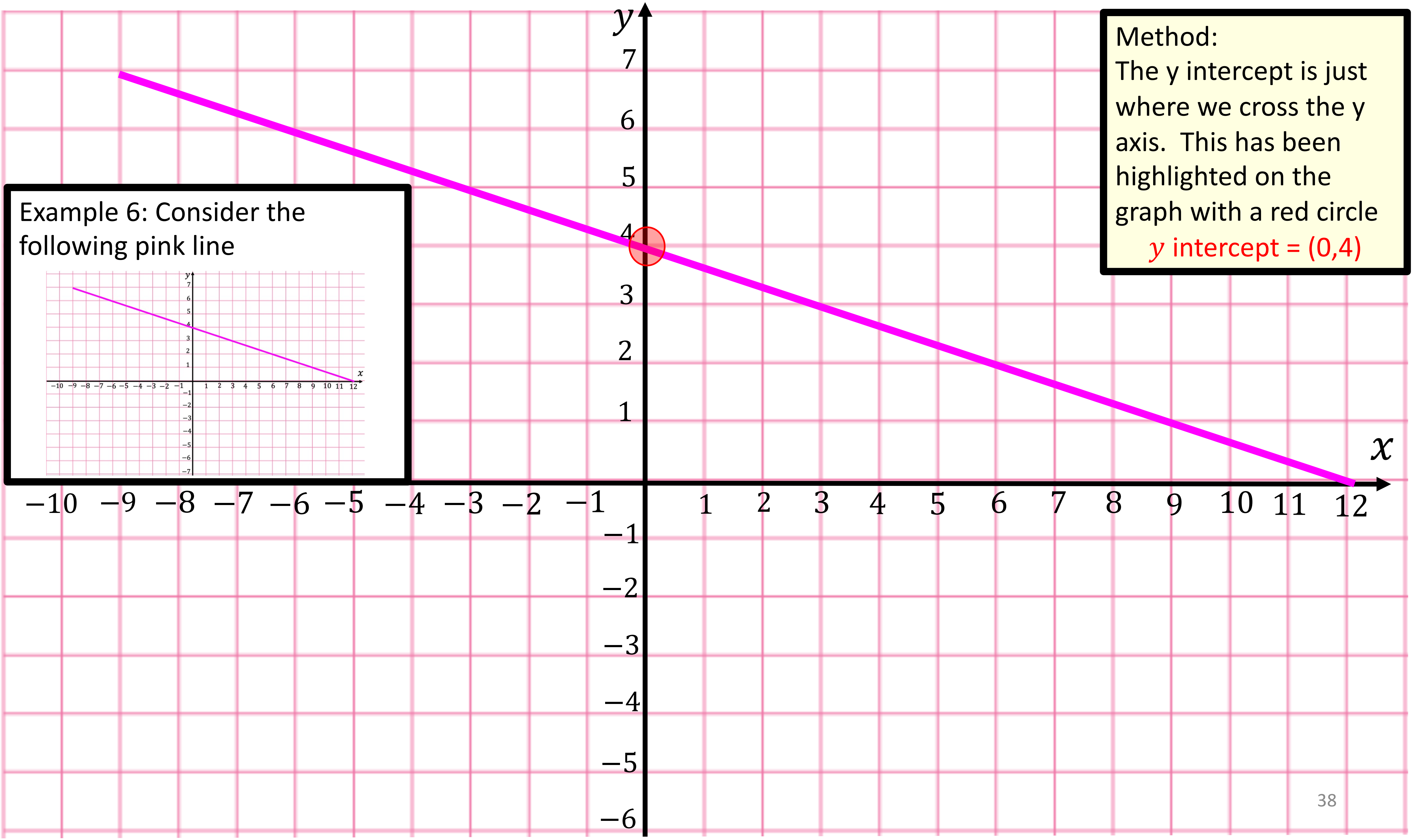




Method:
The y intercept is just where the lines crosses the y axis. When given a graph we can read it off. This has been highlighted on the graph with a red circle.

y intercept = (0,−2)

Example 6: Consider the following pink line



Method:
The y intercept is just where we cross the y axis. This has been highlighted on the graph with a red circle

y intercept = (0,4)

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Way 2:
From An Equation

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The equation of a line looks like $y = mx + c$

The y intercept is represents by the letter c

$$y = mx + c$$

The y intercept is this value here. We use the letter c to represent the y intercept.
Note: some courses use the letter b instead of c to represent the slope

Let's look at some examples

$y = 2x - 1$	$y = -x + 4$	$y = -2 + 3x$	$y = 2 - 4x$	$x = 4$	$y = 5$
y intercept is -1 $(0, -1)$	y intercept is 4 $(0,4)$	Need to re-order this first $y = 3x - 2$ y intercept is -2 $(0, -2)$	Need to re-order this first $y = -4x + 2$ y intercept is 2 $(0,2)$	This is a vertical line since x is the same value the whole time. There is no y intercept	This is a horizontal line since y is the same value the whole time. $y = 5$ is like writing $y = 0x + 5$ y intercept is 5 $(0,5)$

$y + x = 4$	$y - 2x = 5$	$2x + 4y = 5$	$5x - 2y = 7$	$2x + 3y - 1 = 0$	$x + 2y + 5 = 0$
We need to use algebra to re-arrange $y = -x + 4$ y intercept is 4 $(0,4)$	We need to use algebra to re-arrange $y = 2x + 5$ y intercept is 5 $(0,5)$	We need to use algebra to re-arrange $4y = -2x + 5$ $y = \frac{-2x + 5}{4}$ $y = -\frac{1}{2} + \frac{5}{4}$ y intercept is $\left(0, \frac{5}{4}\right)$	We need to use algebra to re-arrange $-2y = -5x + 7$ $y = \frac{-5x + 7}{-2}$ $y = \frac{5}{2}x - \frac{7}{2}$ y intercept is $\left(0, -\frac{7}{2}\right)$	We need to use algebra to re-arrange $3y = -2x + 1$ $y = \frac{-2x + 1}{3}$ $y = -\frac{2}{3}x + \frac{1}{3}$ y intercept is $\left(0, \frac{1}{3}\right)$	We need to use algebra to re-arrange $2y = -x - 5$ $y = \frac{-x - 5}{2}$ $y = -\frac{1}{2}x - \frac{5}{2}$ y intercept is $\left(0, -\frac{5}{2}\right)$

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How Do We Graph An Equation Of A Line?

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Way 1:
Build A Table Of Values

Graph the line $y = 2x - 1$

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Pick x values, let's say -3 to 3 (you are normally given the table with x values already chosen, but if not choose your own and draw out the following table)

x	-3	-2	-1	0	1	2	3
y							

Plug in the x values into the equation $y = 2x - 1$ in order find the y values (replace every x value in the equation)

x	-3	-2	-1	0	1	2	3
y	$2(-3) - 1$	$2(-2) - 1$	$2(-1) - 1$	$2(0) - 1$	$2(1) - 1$	$2(2) - 1$	$2(3) - 1$

Simplify each y

x	-3	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3	5

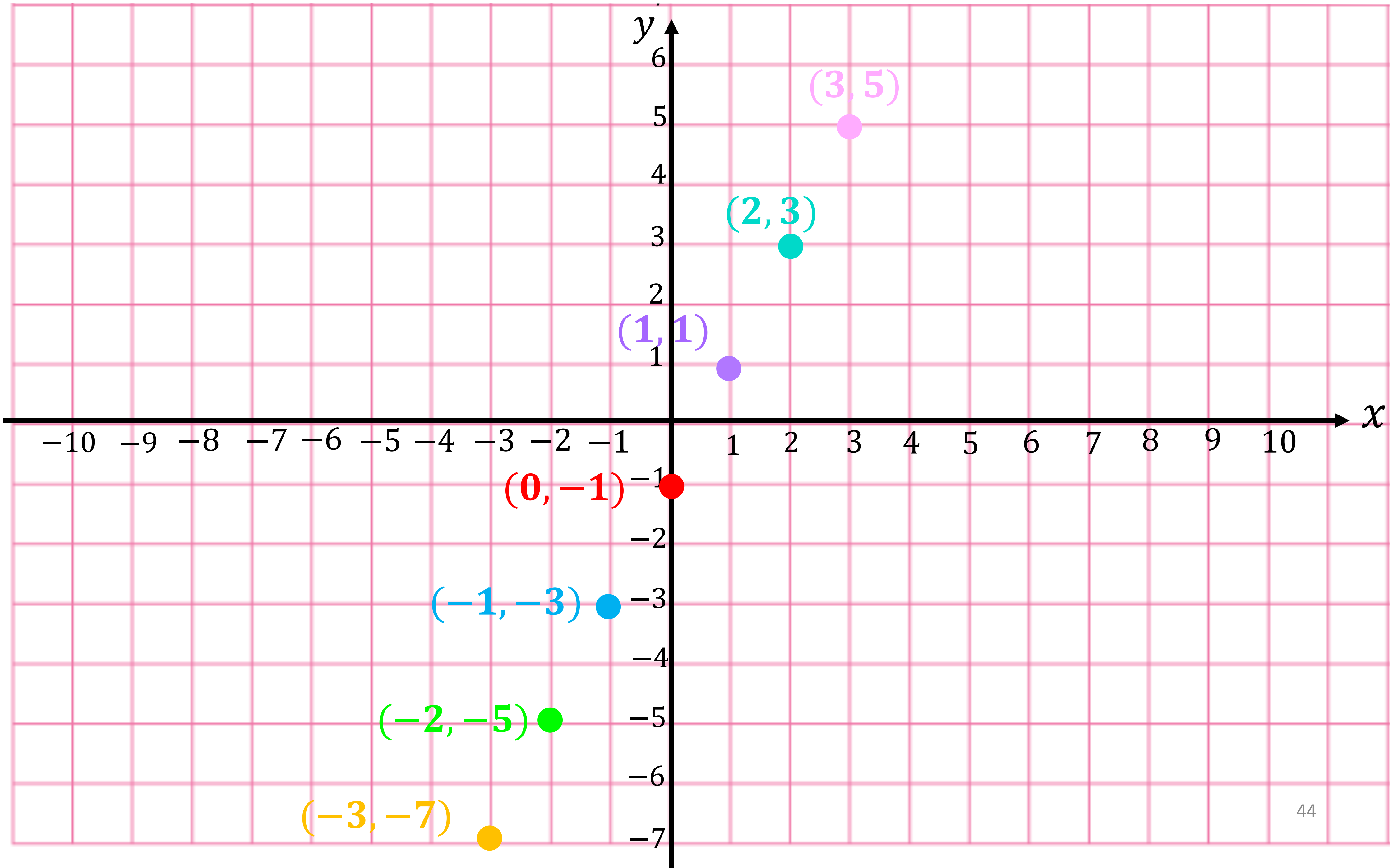
Let's colour code each coordinate

x	-3	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3	5

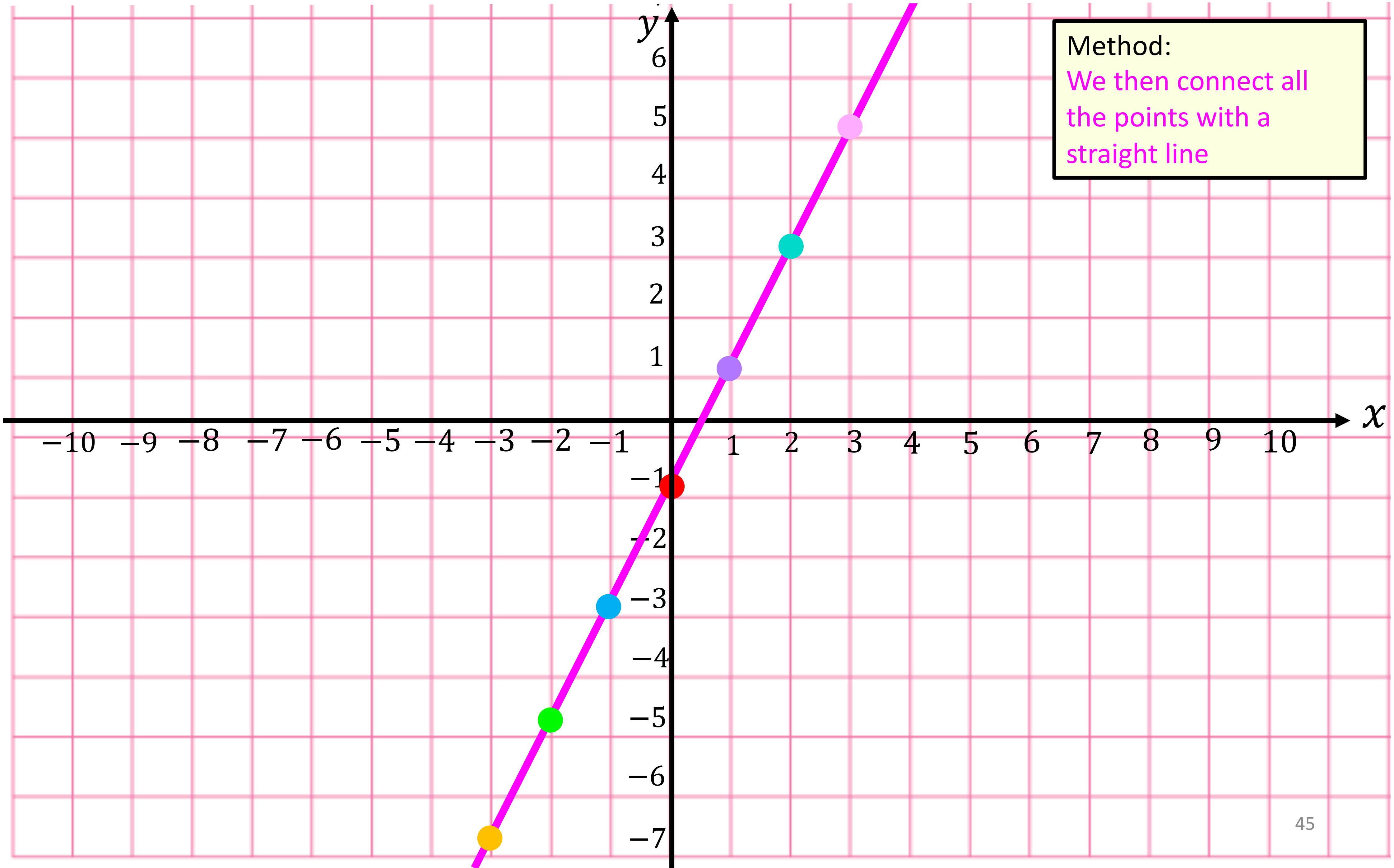
Note: Harder questions don't always give the line in the form $y = mx + c$. We need to use algebra to make y the subject in order to get into the form $y = mx + c$ first before building the table of values.

Plot each pair of points (each colour pair). We will do this on the next page

$(-3, -7)$ $(-2, -5)$ $(-1, -3)$ $(0, -1)$ $(1, 1)$ $(2, 3)$ $(3, 5)$



y
x
6
5
4
3
2
1
−1
−2
−3
−4
−5
−6
−7
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8 9 10
(3, 5)
(2, 3)
(1, 1)
(0, −1)
(−1, −3)
(−2, −5)
(−3, −7)



Method:
We then connect all the points with a straight line

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Way 2:

Start with the y intercept and move by the gradient

Before we start, the following can help to remember what we are about to learn :

$$y = mx + c$$

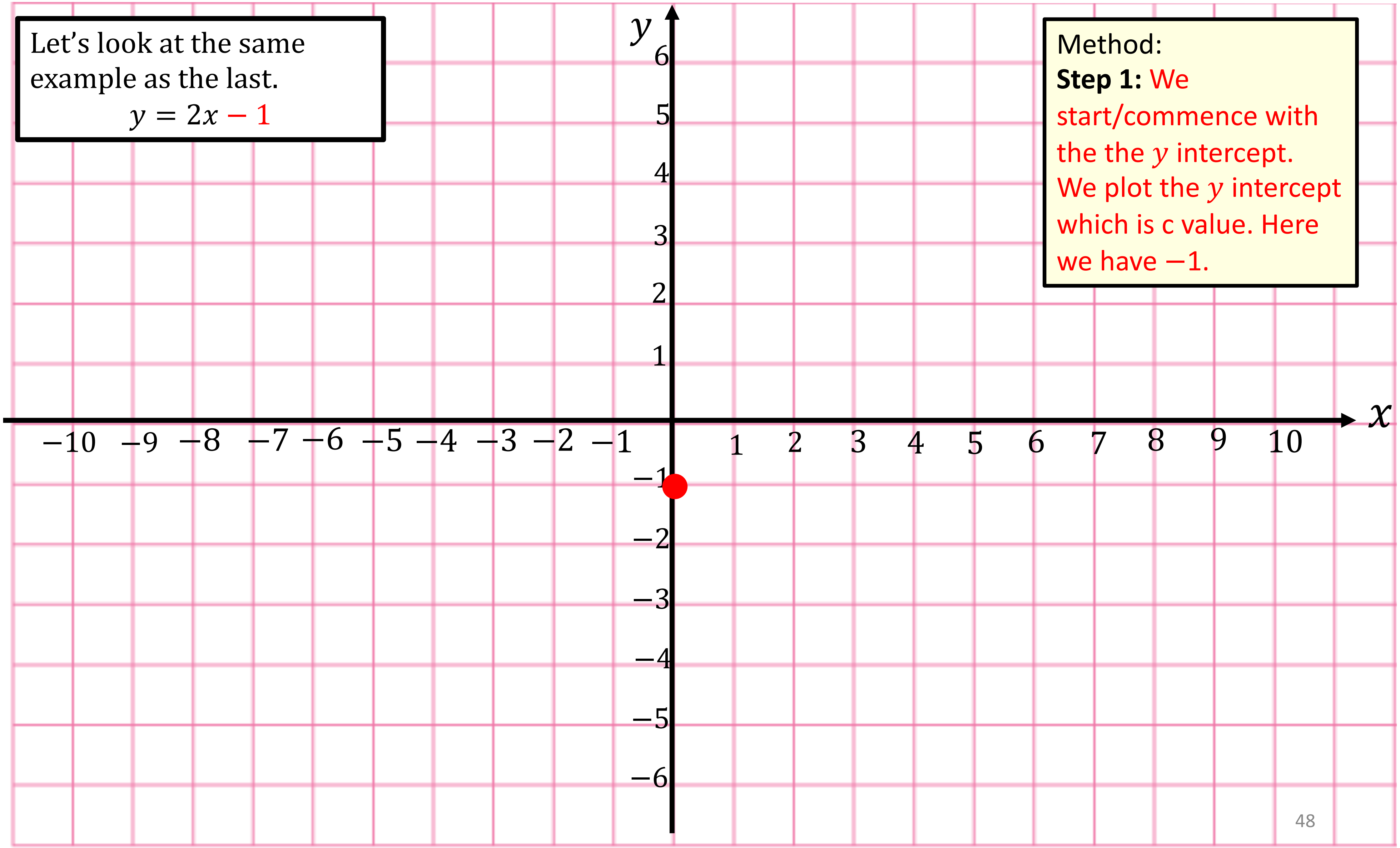
M O V E

C O M M E N C E

The c value is where we **commence** and the value m is **how we move** to the next point on the graph

Let’s look at the same example as the last.

$$y = 2x - 1$$

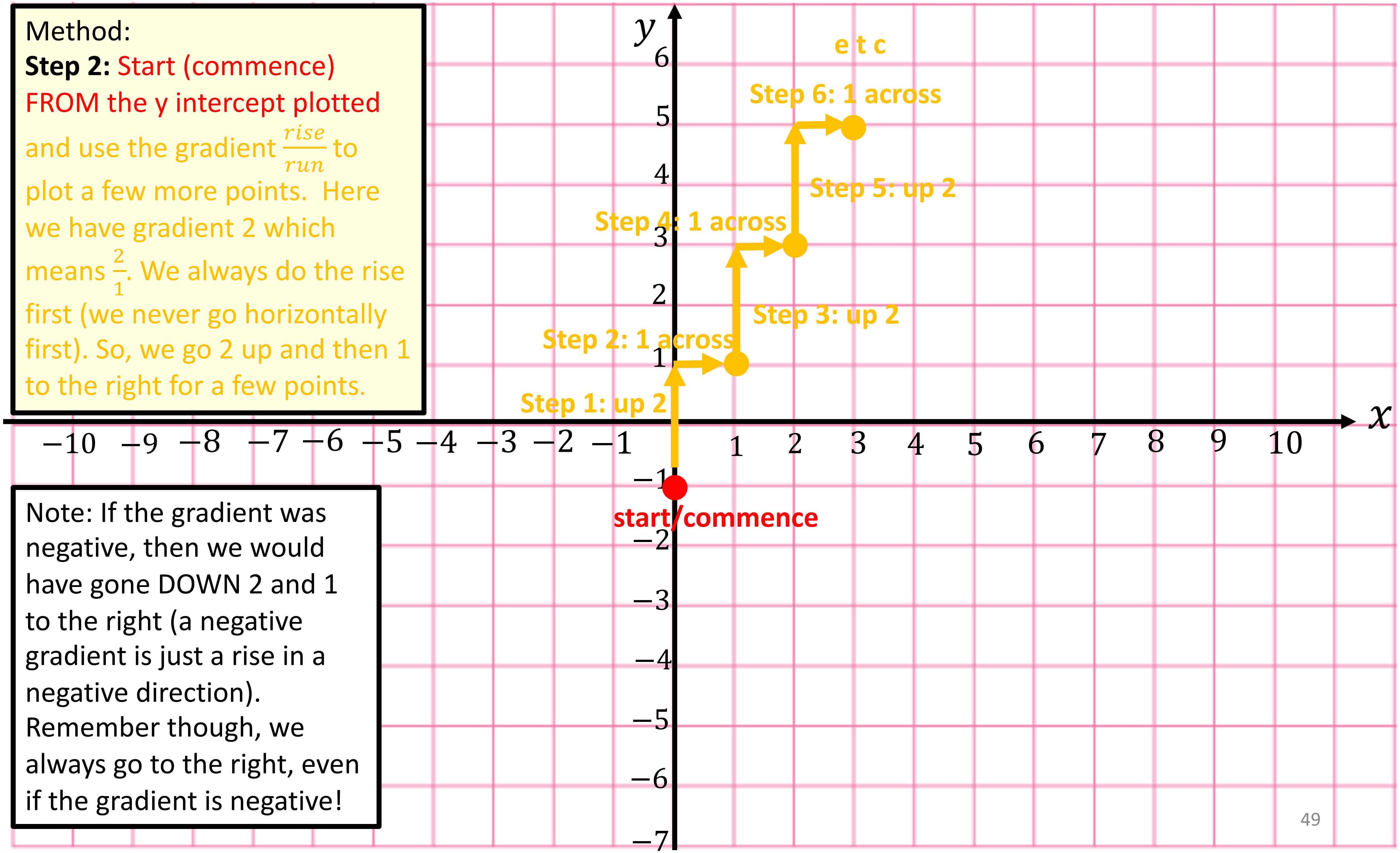


Method:

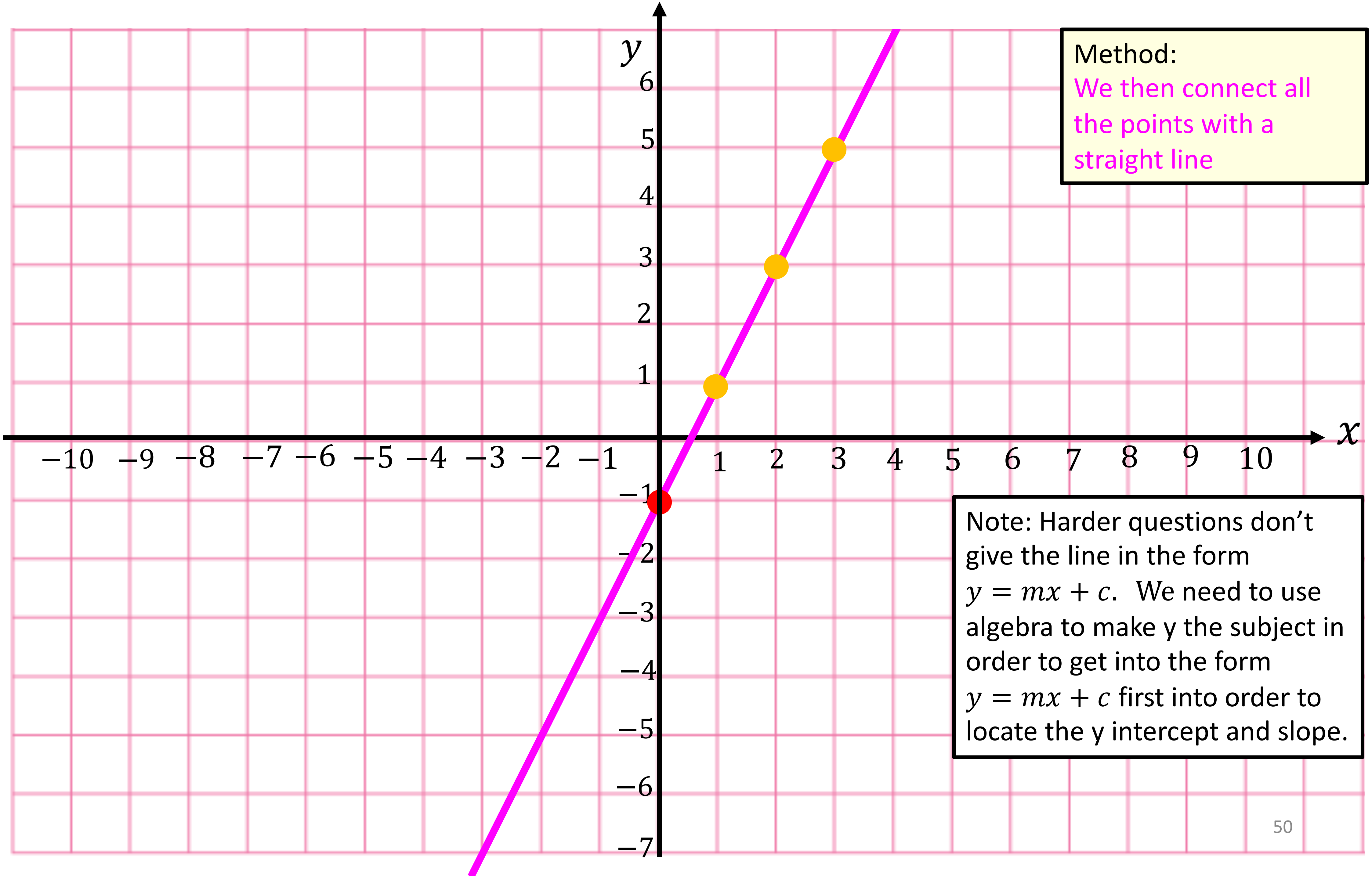
Step 1: We start/commence with the the y intercept. We plot the y intercept which is c value. Here we have -1.

Method:

Step 2: Start (commence) FROM the y intercept plotted and use the gradient $\frac{rise}{run}$ to plot a few more points. Here we have gradient 2 which means $\frac{2}{1}$. We always do the rise first (we never go horizontally first). So, we go 2 up and then 1 to the right for a few points.



Note: If the gradient was negative, then we would have gone DOWN 2 and 1 to the right (a negative gradient is just a rise in a negative direction). Remember though, we always go to the right, even if the gradient is negative!



Method:
We then connect all the points with a straight line

Note: Harder questions don't give the line in the form $y = mx + c$. We need to use algebra to make y the subject in order to get into the form $y = mx + c$ first into order to locate the y intercept and slope.

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Way 3:
Find two coordinates, plot them and "connect the dots"

A line is defined by two points. If we have two points, then we can connect the points just like "connecting the dots" and create the line. What points should be pick? Zero is a really easy number isn't it, so let's try $x = 0$ and $y = 0$.

For example, graph the line $y = 2x - 6$

Let $x = 0$

$x = 0$ means we replace x with 0 in the equation $y = 2x - 6$

$$y = 2(0) - 6$$

We now need to solve for y. This is easy since y is already on its own

$$y = 0 - 6$$

$$y = -6$$

So, we have the point $(0, -6)$

Let $y = 0$

$y = 0$ means we replace y with 0 in the equation $y = 2x - 6$

$$0 = 2x - 6$$

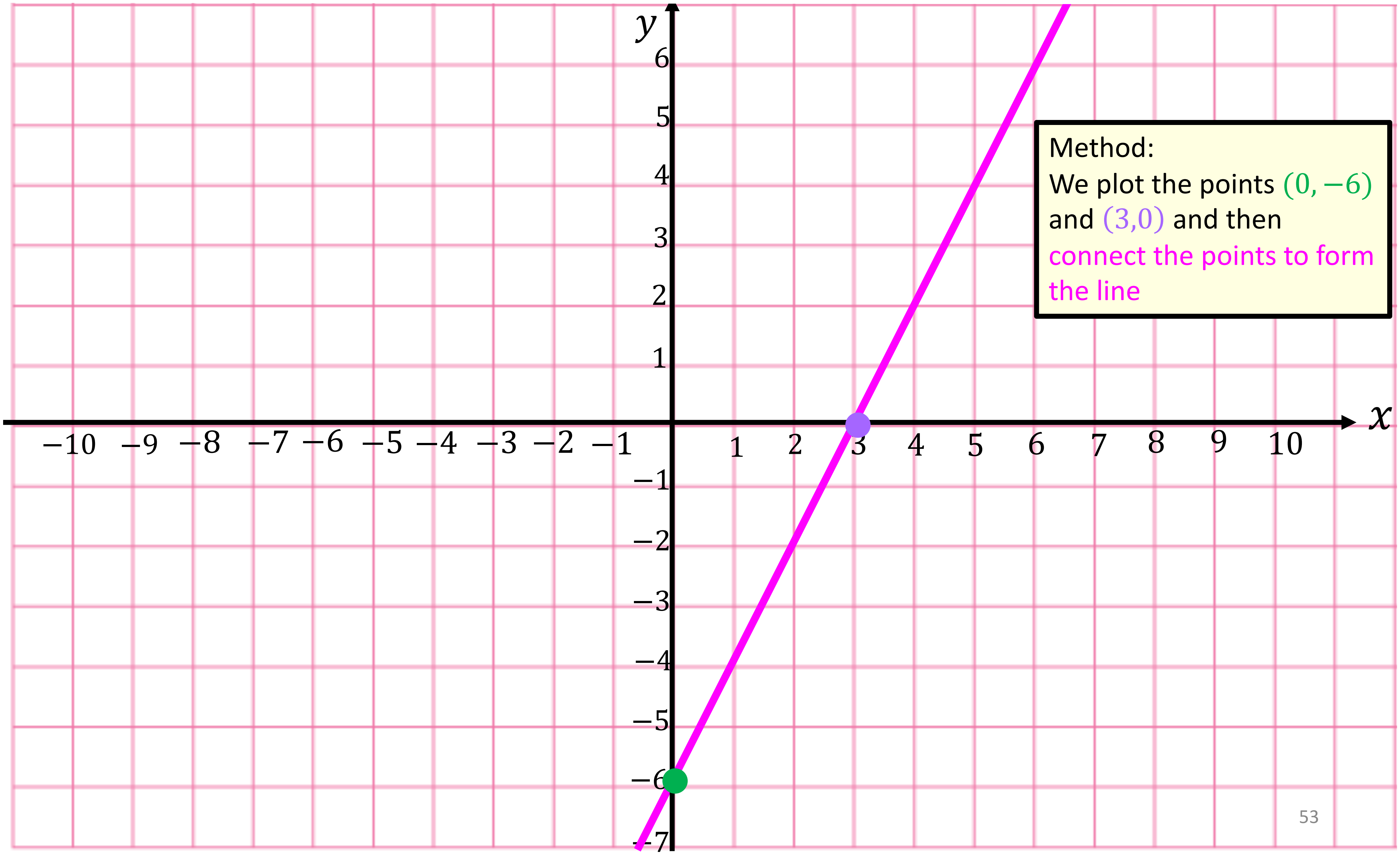
We now need to solve for x. This time we need to re-arrange to find x using algebra as it is not already on its own

$$2x = 6$$

$$x = 3$$

So, we have the point $(3,0)$

$(0, -6)$ and $(3,0)$ give us two points that define the line. To graph the line, let's now plots the 2 points and connect them.

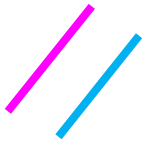


Method:
We plot the points $(0, -6)$ and $(3,0)$ and then connect the points to form the line

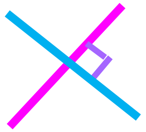
What Are Parallel And Perpendicular Lines?

Parallel lines the lines have the **same** gradient . They never meet

For example, if one line has a slope of 2 then a parallel line will also have a slope of 2.



Perpendicular lines meet at right angles. This means the slopes multiply to make -1 or they are negative reciprocals of each other. The easiest way to find the **negative reciprocal** is to simply flip the fraction and change the sign (a positive gets changed to a negative and a negative gets changed to a positive). For example, if one line has a slope of 2 then a perpendicular line will have a slope of $-\frac{1}{2}$.



Notice how the signs of the gradient change since one gradient will be positive and one will be negative

Let's look at some examples for perpendicular lines as this is a hard concept for some students.

- If a line has slope 2, what slope would a perpendicular line have?
  slope 2 means the same thing as $\frac{2}{1}$. Flipping the fraction gives $\frac{1}{2}$. Changing the sign means we have a negative, so $-\frac{1}{2}$. Hence a perpendicular line has slope $-\frac{1}{2}$.
  Let's check if we have done this correctly by checking if the slopes multiply to make -1:
  $$2\left(-\frac{1}{2}\right) = -1.$$ Yes, they do, as we expected!
- If a line has slope $-\frac{2}{3}$, what slope would a perpendicular line have?
  Flipping the fraction gives $\frac{3}{2}$. Changing the sign means we have a positive. Hence a perpendicular line has slope $\frac{3}{2}$.
  Let's check if we have done this correctly by checking if the slopes multiply to make -1:
  $$-\frac{2}{3}\left(\frac{3}{2}\right) = -1.$$ Yes, correct again!
- If a line has slope $\frac{1}{3}$, what slope would a perpendicular line have?
  Flipping the fraction gives $\frac{3}{1}$. Changing the sign means we have a negative so $-\frac{3}{1}$. Hence a perpendicular line has slope $-\frac{3}{1}$ which is just -3.
  Let's check if we have done this correctly by checking if the slopes multiply to make -1:
  $$\frac{1}{3}(-3) = -1.$$ Yes, correct again!

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How Do We Find The Equation Of A Line?

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The equation of a straight line looks like

$$y = mx + c$$

Recall that we use the letter m for gradient/slope and the letter c for y intercept

$$y = mx + c$$

gradient/slope y intercept

So, we just need to find the gradient/slope m and y intercept c and then we are done!

For the next 2 pages concentrate on all examples in this left column first

$y = mx + c$

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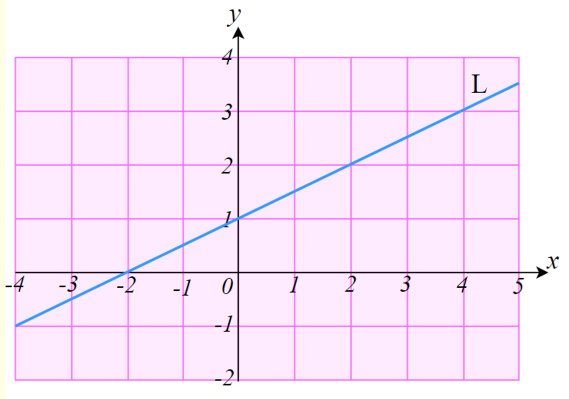
Next concentrate on this right column

Step 1: Find m

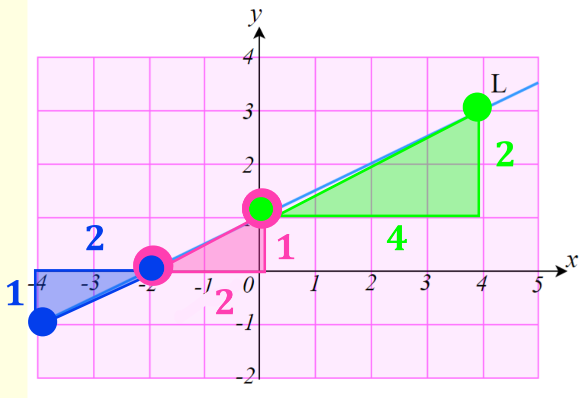
There are 4 ways to find this dependent on what we're given

Type 1: If given graph - pick any 2 points on the line, form a triangle & work out the $\frac{rise}{run}$

e.g. Find the slope of the following line below on the graph on the left



Solution $\Longrightarrow$
pick a pair of points and form any triangle (above or below the line) and work out the $\frac{rise}{run}$



It doesn't matter which triangle we build (all give the same answer -). Let's use all 2 triangles formed above.

$$\frac{\text{rise}}{\text{run}} = \frac{1}{2} \quad \text{or} \quad \frac{1}{2} \quad \text{or} \quad \frac{2}{4} = \frac{1}{2}$$

The slope is positive is the line is going up from left to right (rise) and negative if the line is going down from left to right, so we know that have a positive slope.

$$y = \frac{1}{2}x + c$$

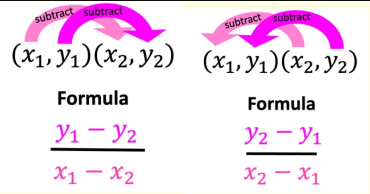
Alternative method: we can just write down any 2 points ("nice points" that are whole numbers) from the graph and proceed as in way 2 below

Type 2: If given 2 points - use the following slope formula:

e.g. Find the equation of the line passing through the points $(-1,3)$ and $(2,4)$

$$m = \frac{4-3}{2--1} = \frac{1}{3} \qquad \text{or} \qquad m = \frac{3-4}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$$

$$y = \frac{1}{3}x + c$$

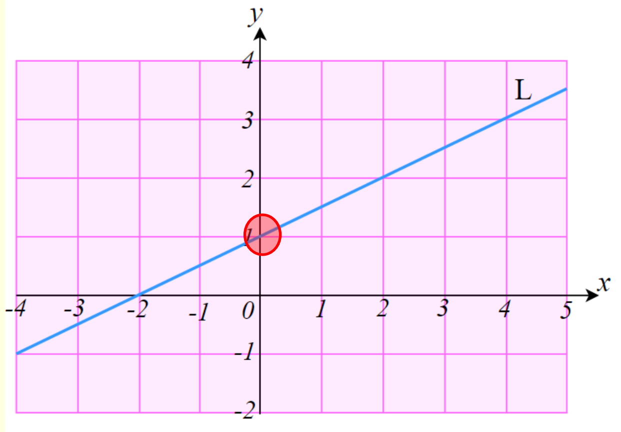


Step 2: Find c

There are 2 ways to find this dependent on what we're given

Type 1: If given the graph -
c is just the value where the graph crosses the y axis. We can read this off easily.

e.g. Find the y intercept of the following line



using step 1 we know we have $y = \frac{1}{2}x + c$

We can see that c is 1 from the graph (red circle)

$$y = \frac{1}{2}x + 1$$

Type 3: If given a line that parallel to – locate slope and use same slope

e.g. 1 Find the line parallel to $y = 2x - 3$

$y = 2x - 3$ has gradient 2. Since parallel means the same gradient, we use the same gradient 2.

$$y = 2x + c$$

e.g. 2 Find the line parallel to $6x + 2y = 5$

we must first re-arrange using algebra to get into the form $y = mx + c$. We do this in order to spot the gradient.

$$2y = -6x + 5$$
$$y = \frac{-6x + 5}{2}$$
$$y = -3x + \frac{5}{2}$$

$y = -3x + \frac{5}{2}$ has gradient -3 . Since parallel means the same gradient, we use the same gradient -3.

$$y = -3x + c$$

Type 4: If given a line perpendicular to – locate slope and use negative reciprocal slope (negative reciprocal means we flip the fraction and change the sign)

e.g. 1 Find the line perpendicular to $y = 2x - 3$

$y = 2x - 3$ has gradient 2 . Since perpendicular means the negative reciprocal gradient $-\frac{1}{2}$

$$y = -\frac{1}{2}x + c$$

e.g. 2 Find the line perpendicular to $4x + 2y = 6$

we must first re-arrange using algebra to get into the form $y = mx + c$. We do this in order to spot the gradient.

$$2y = -6x + 5$$
$$y = \frac{-6x + 5}{2}$$
$$y = -3x + \frac{5}{2}$$

$y = -3x + \frac{5}{2}$ has gradient -3. Since perpendicular means the negative reciprocal gradient, we use the negative reciprocal $\frac{1}{3}$

$$y = \frac{1}{3}x + c$$

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Type 2: If given a point passes through - plug in the point since the point (x, y) tells us what x and y are. Then solve for c using algebra.

e.g. Find the line parallel to $y = 2x - 3$ and passing through $(-1,4)$

using step 1 (way 3) we know we have slope 2 hence $y = 2x + c$

Now we plug in the point $(-1 , 4)$ into $y = 2x + c$.
This means we replace x with -1 and y with 4 and solve for c

$$4 = 2(-1) + c$$

Solve for c using algebra

$$4 = -2 + c$$
$$c = 4 + 2 = 6$$
$$y = 2x + 6$$

Note:
If given 2 points that the line passes through, choose either point to plug in. Both will give the same answer for c.

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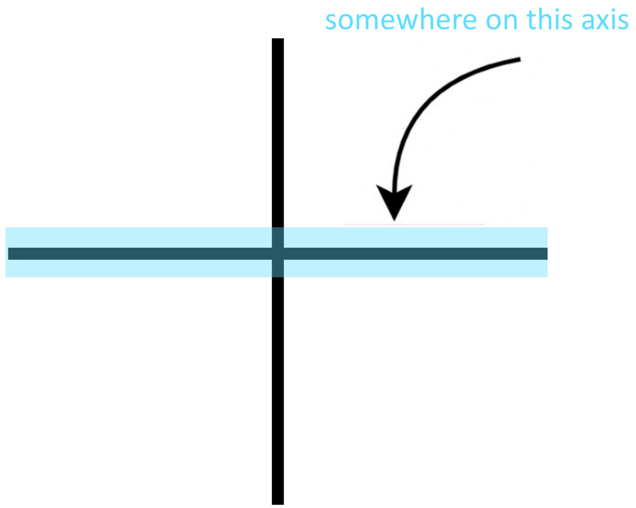
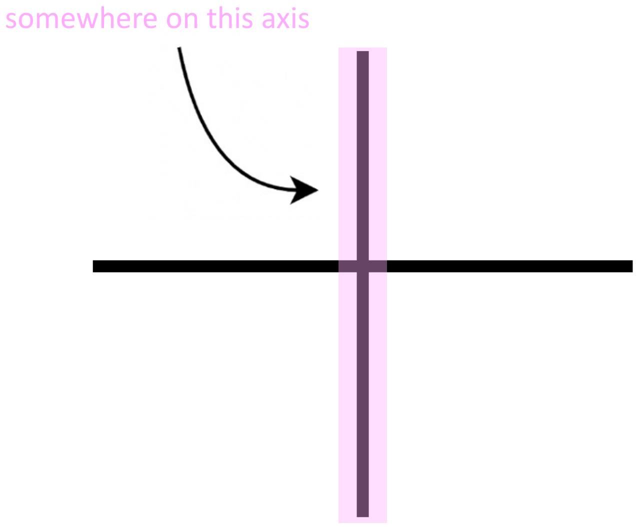
How Do We Find x and y Intercepts When Given An Equation In Any Form?

The x intercept is the point where the graph crosses the x axis and the y intercept is the point where the graph crosses the y axis

x ***intercept***	y ***intercept***
	
To find this point we set $y = 0$ (i.e. replace y with 0) and solve for x. The coordinate will be $(x, 0)$ where x is the value found.	To find this point we set $x = 0$ (i.e. replace x with 0) and solve for y. The coordinate will be $(0, y)$ where y is the value found. Remember that we can also just quickly read this value off from an equation. It is the c value.

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How Do We Find Midpoints?

Midpoint between 2 points $(x_1, y_1), (x_2, y_2) \Rightarrow \text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

In English this formula just says:

Add the x coordinates and divide by 2 (i.e. find the average) and add the y coordinates and divide by 2 (i.e. find the average)

Examples To Try

Find the midpoint between the two points

i. $(1,4)$ and $(2,8)$

ii. $(-2,3)$ and $(4, -1)$

Harder Examples To Try

iii. The midpoint of two points $(a, 6)$ and $(7,10)$ is $(3,9)$. Find the value of a

iv. The midpoint joining the two points $(5,9)$ and (a, b) is $(8,1)$. Find the values of a and b

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How Do We Find Distances?

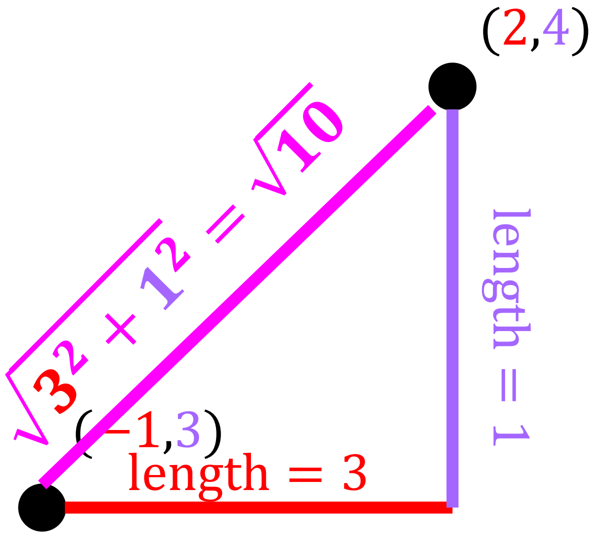
There are 2 ways to find the distance:

Way 1: **Build A Triangle** - We find the x and y distances between the coordinates and use Pythagoras to find the hypotenuse length which is the distance between the points

Way 2: **Formula** - Distance between 2 points $(x_1, y_1), (x_2, y_2) \Longrightarrow \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example: Find the distance between the 2 points $(-1,3)$ and $(2,4)$
Let's colour code as $(-1, 3), (2,4)$

Way 1: Build a triangle



Distance= $\sqrt{3^2 + 1^2} = \sqrt{10}$

Way 2: Distance Formula

$$= \sqrt{(2 - -1)^2 + (4 - 3)^2}$$
$$= \sqrt{3^2 + 1^2}$$
$$= \sqrt{10}$$

Examples To Try

Find the distance between the two points

i. (2,3) and (3,6)

ii. (2,5) and $(-1,3)$

iii. The distance between two points $(a, 3)$ and $(5,7)$ is 5. Find the value(s) of a

A triangle has vertices P, Q and R

The coordinates of P are $(-3,-6)$

The coordinates of Q are (1,4)

The coordinates of R are $(5,-2)$

M is the midpoint of PQ

N is the midpoint of QR

Prove that MN is parallel to PR

The coordinates of three points are A$(-4,-1)$ B(8,9) and C(k,7). M is the midpoint of AB and MC is perpendicular to AB. Find the value of k.

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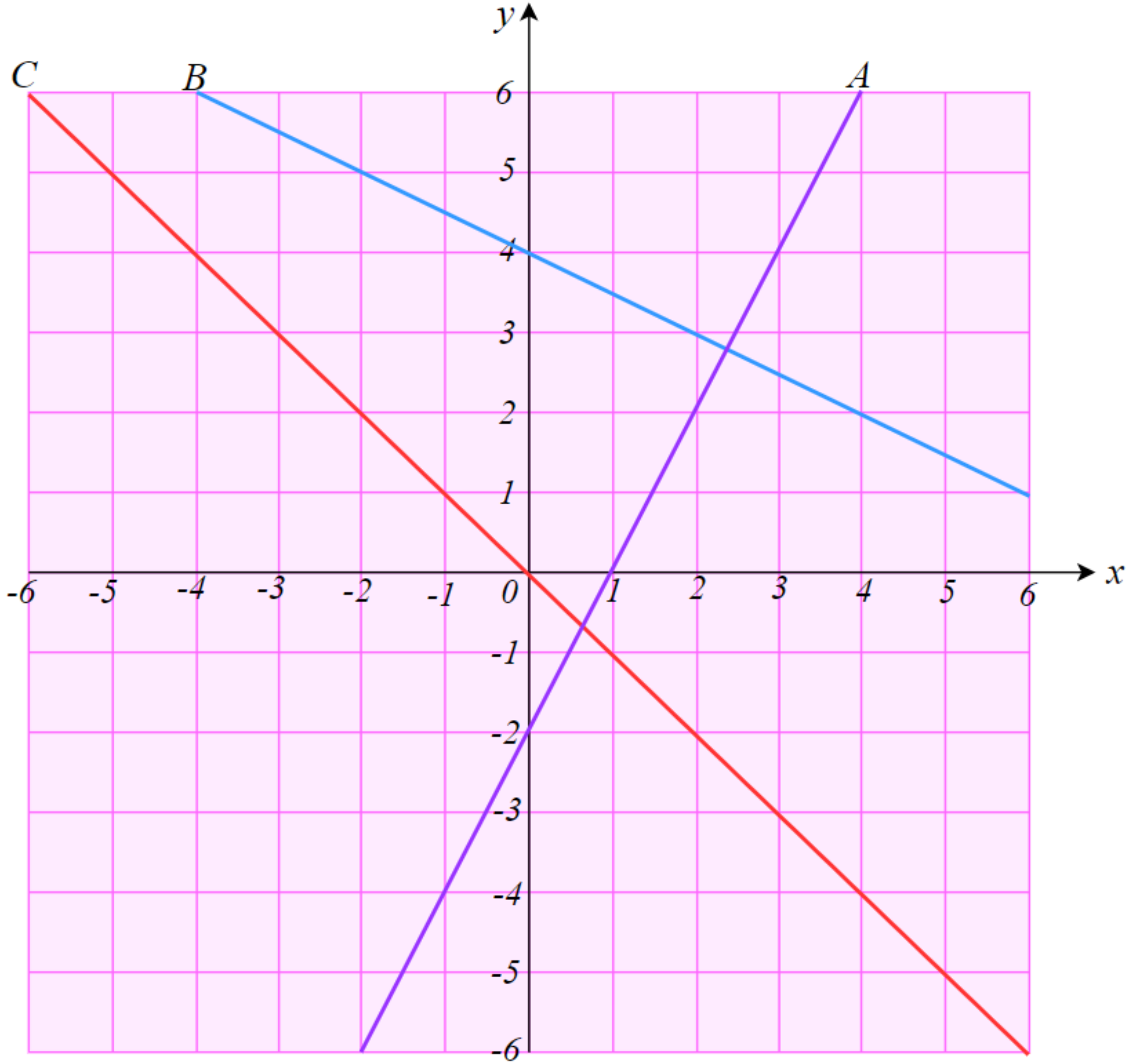
Harder Examples Of Each Type

Level 1: Bronze

Find the equations of the following 3 lines



Level 2: Silver

A is the point (0,1). B is the point (10,6)

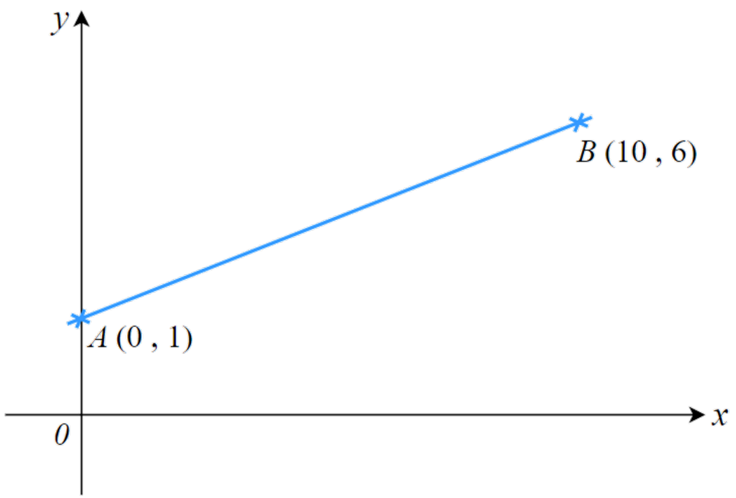
The equation of the straight line through A and B is $y = \frac{1}{2}x + 1$

i. Write down an equation of another straight line that is parallel to $y = \frac{1}{2}x + 1$

ii. Write down an equation of another straight line which passes through the point (0,1)

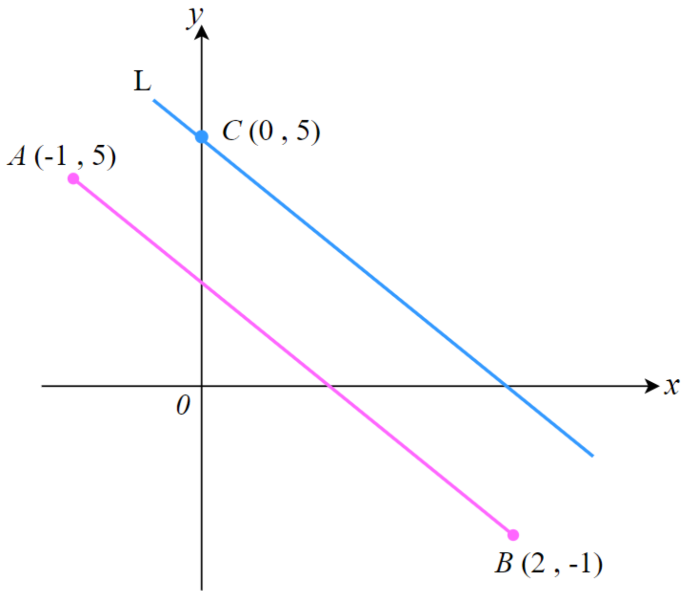
iii. Find the equation of another straight line which is parallel to $y = \frac{1}{2}x + 1$ and passes through the point (2,5)

iv. Find the equation of the line perpendicular to AB passing through B



The diagram shows three points A(1,−5), B(2,−1) and C(0,5)

The line L is parallel to AB and passes through C. Find the equation of the line L.



Find an equation of the line joining A (7, 4) and B (2, 0), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Level 3: Gold

Here are the equations of four straight lines

Line A: $y = 2x + 4$

Line B: $2y = x + 4$

Line C: $2x + 2y = 4$

Line D: $2x - y = 4$

Two of these lines are parallel. Which 2 lines?

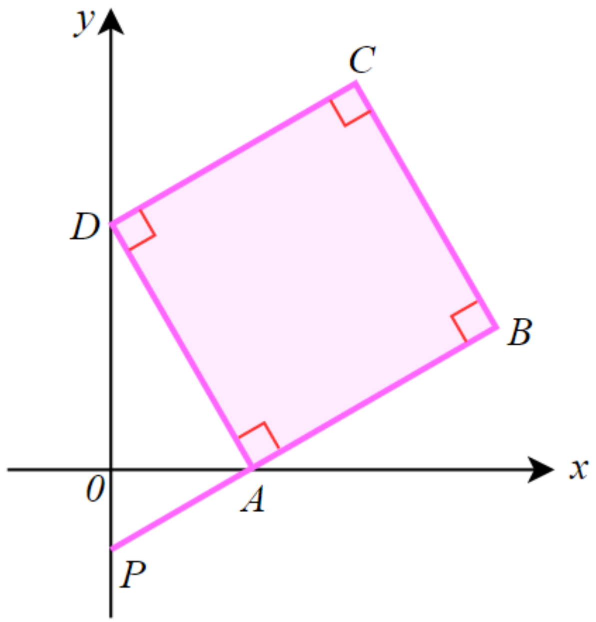
P has coordinates $(-9, 7)$. Q has coordinates $(11,12)$. M is the midpoint of the line segment PQ. Line L is perpendicular to the line segment PQ. L passes through M. Find an equation for L.

ABCD is a kite with AB=AD and CB=CD. B is the point with the coordinates (10,19). D is the point with the coordinates (2,7).
Find the equation of the line AC in the form $px + qy = r$, where p, q and r are integers
Hint: The diagonals of a kite and perpendicular. We need to find equation of line perpendicular to BD passing through midpoint of BD.

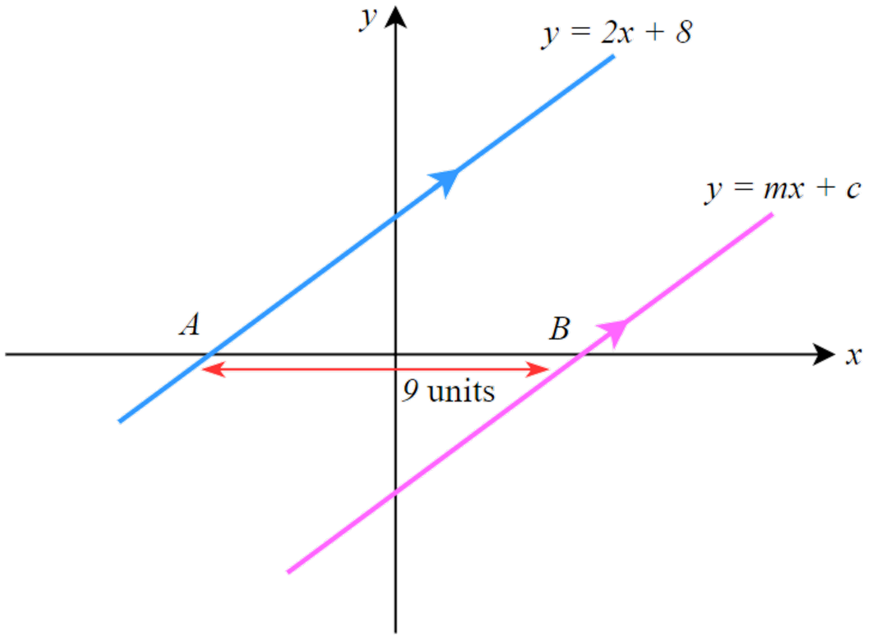
A has coordinates $(-3,0)$
B has coordinates (1,6)
C has coordinates (5,2)
Find the equation of the line passing through C that is perpendicular to AB. Give your equation in the form $ax + by = c$ where a, b and c are integers

The point P has coordinates (3,4)
The point Q has coordinates (a, b)
A line perpendicular to PQ is given by the equation $3x + 2y = 7$
Find an expression of b in terms of a

ABCD is a square. P and D are points on the y axis. A is a point on the *x* axis. PAB is a straight line. The equation of the line that passes through the points A and D is $y = -2x + 6$ Find the length of PD.



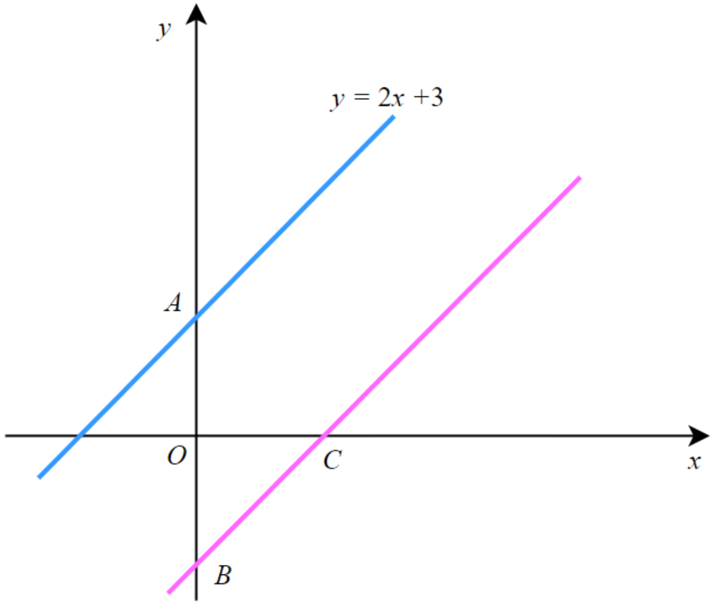
The line $y = mx + c$ is parallel to the line $y = 2x + 8$. Find the value of m and the value of c



The distance AB is 7 units.

i. Write down the equation of the line through B which is parallel to $y = 2x + 3$

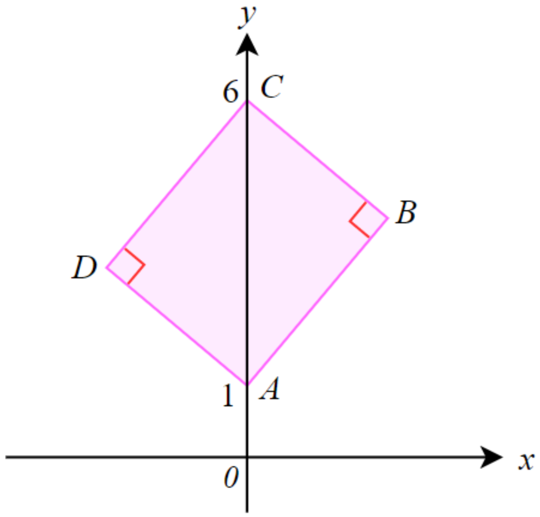
ii. Find the coordinates of the point C where this line crosses the x axis



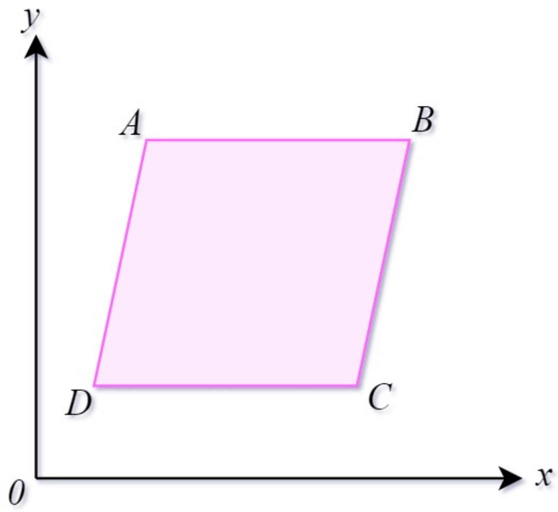
Level 4: Diamond

ABCD is a rectangle. A is the point (0,1). C is the point (0,6). The equation of the straight line through A and B is $y = 2x + 1$

- Find the equation of the straight line through D and C
- Find the equation of the straight line through B and C



ABCD is a rhombus. The coordinates of A are (5,11). The equation of the diagonal DB is $y = \frac{1}{2}x + 6$. Find the equation of the diagonal AC.



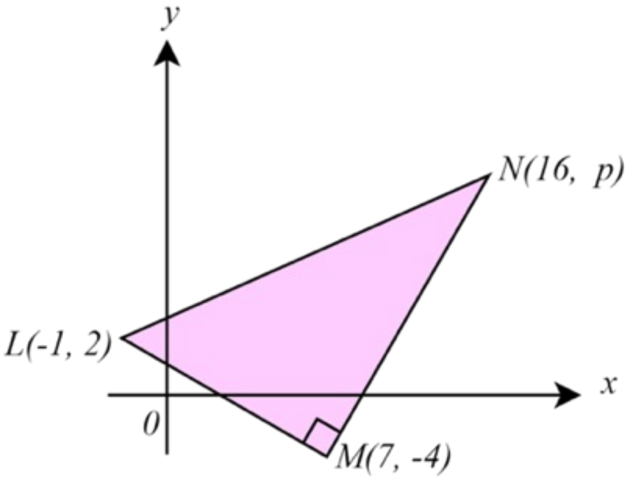
The figure above shows a right-angled triangle LMN. The points L and M have coordinates (–1, 2) and (7, –4) respectively

i. Find an equation for the straight line passing through the points L and M. Give your answer in the form $ax+by+c = 0$, where a, b and c are integers

Given that the coordinates of point N are (16, p), where p is a constant, and angle LMN = 90°

ii. find the value of p

iii. Given that there is a point K such that the points L, M, N, and K form a rectangle, find the *y* coordinate of K.



Triangle HJK is isosceles with HJ=HK and JK= $\sqrt{80}$

H is the point with coordinates $(-4,1)$

J is the point with coordinates $(j, 15)$ where $j < 0$

K is the point with coordinates $(6, k)$

M is the midpoint of J

The gradient of HM is 2

Find the value of j and the value of k

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How Do We Find Where 2 Lines Intersect?

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If Given Graph

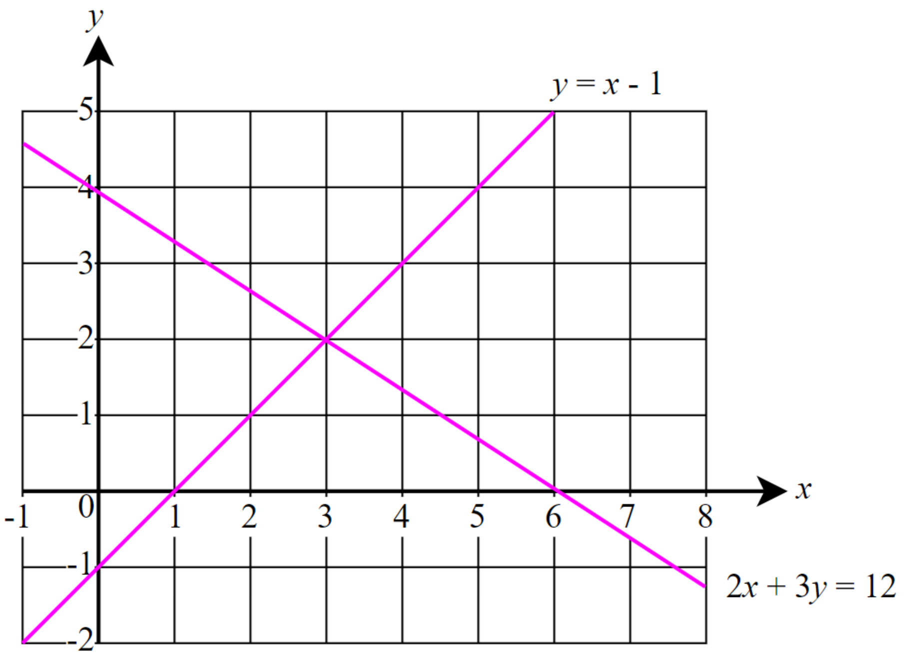
The diagram shows two straight lines.

The equation of the lines are

$$y = x - 1 \; and \; 2x + 3y = 12$$

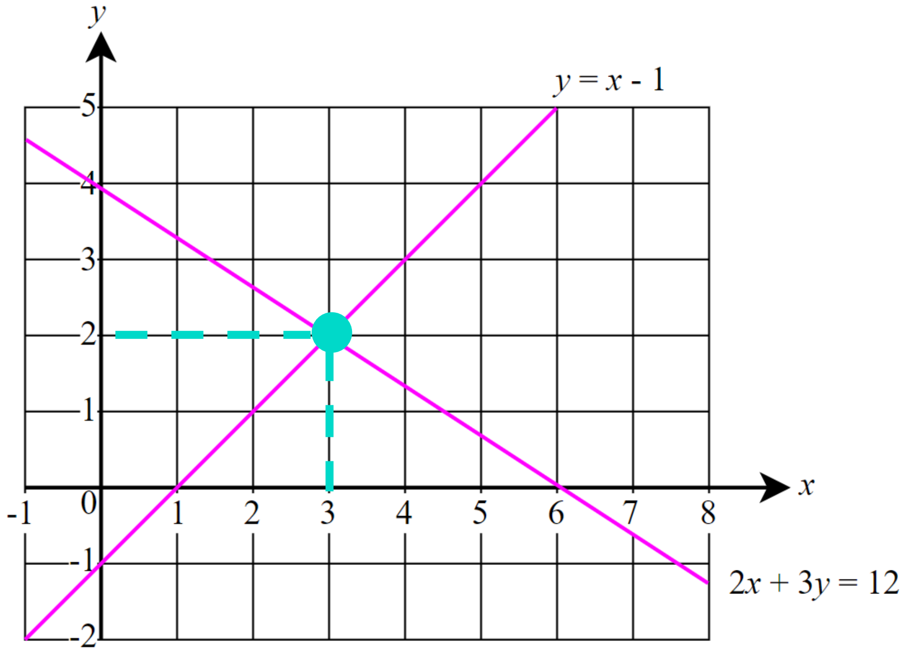
Write down the solution of the simultaneous equations

$$y = x - 1$$
$$2x + 3y = 12$$



The solution is just the point where the graphs intersect!

$$x = 3$$
$$y = 2$$



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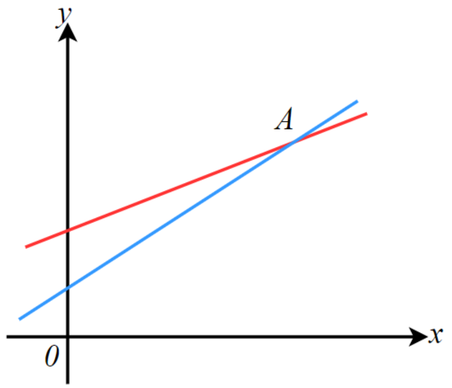
If Given Equations

The equation of two straight lines are

$$y = 2x + 7$$
$$y = 3x + 4$$

Find the coordinates where these lines intersect



We just **solve simultaneously**! Solving simultaneously finds the intersection point.
You can either use elimination or substitution. These methods won't be covered here (see the relevant notes and worksheet for this topic)

Finding the intersection points (A and B) is just solving simultaneously

$$y = 2x + 7 \text{ and } y = 3x + 4$$

Both equations are already re-arranged for y, so setting them equal

$$2x + 7 = 3x + 4$$

Now solving for x

$$x = 3$$

Subbing into one of the original equations

$$y = 2x + 7$$
$$y = 2(3) + 7$$
$$y = 13$$

The graphs intersect at (3,13)

$$x = 3, y = 13$$

Examples To Try

The equation of two straight lines are

$$y = 2x - 3$$
$$y = x - 6$$

Find the coordinates where they intersect

The equation of two straight lines are

$$y = 3x + 4$$
$$2y = 6x + 4$$

Find the coordinates where they intersect

The equation of two straight lines are

$$8x - 3y = -2$$
$$y = 3 - 2x$$

Find the coordinates where they intersect

The equation of two straight lines are

$$3x + 2y = 4$$
$$4x + 5y = 17$$

Find the coordinates where they intersect

Harder Examples To Try

A and B are lines
Line A has equation $2y = 3x + 8$
Line B goes through the points $(-1,2)$ and $(2,8)$
Do lines A and B intersect?

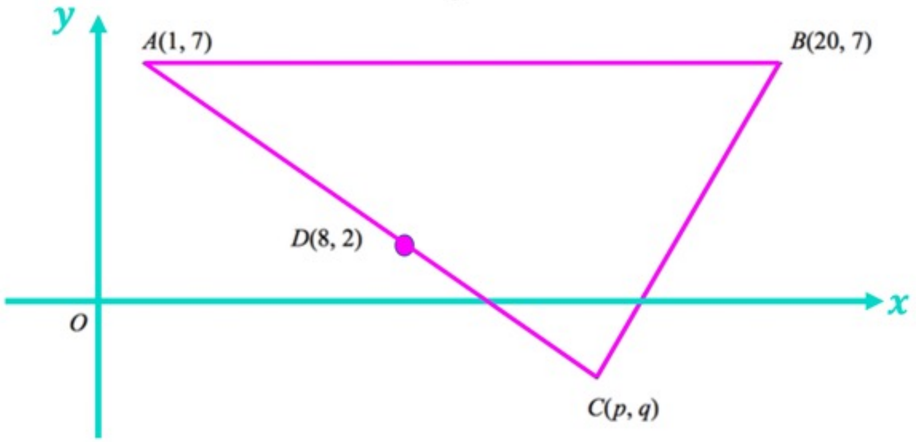
The straight line L_1 passes through the points with coordinates (4,6) and (12,2)
The straight line L_2 passes through the origin and has gradient -3
The lines L_1 and L_2 intersect at point P. Find the coordinates of P

The points A(1,7), B(20,7) and C(p, q) form the vertices of a triangle ABC as show in the diagram. The point D(8,2) is the midpoint of AC.

i. Find the values of p and q

The line l_1 which passes through D and is perpendicular to AC, intersects AB at E

i. Find the equation for l, in the form $ax + by + c = 0$, where a, b and c are integers
ii. Find the exact x coordinate of E



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Areas

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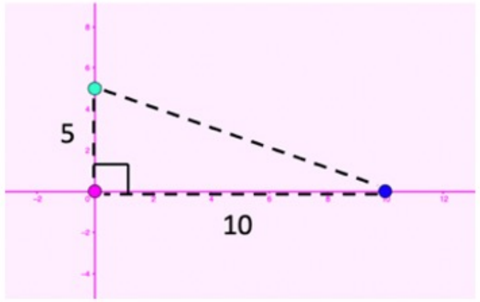
If given coordinates, we can find the areas of shapes

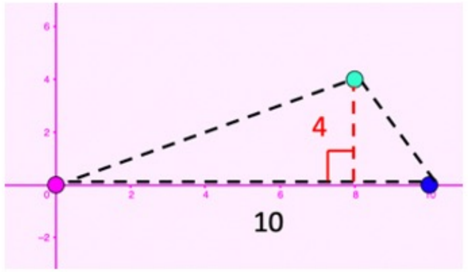
Triangle OQR given the coordinates OQR 0(0,0) Q(10,0) R(0,5)



$$\text{Area} = \frac{1}{2} \times base \times height$$
$$= \frac{1}{2} \times 10 \times 5$$
$$= 25$$

Triangle OQP given by the coordinates 0(0,0), Q(10,0) P(8,4)

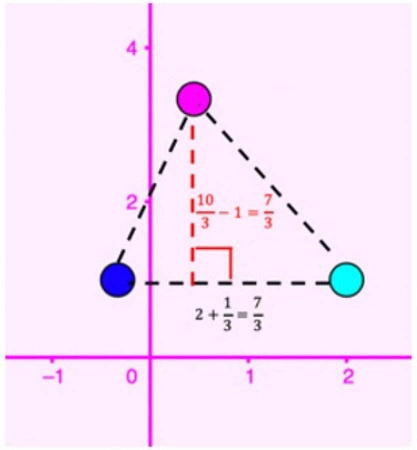
We need to form the red line to see the height of the triangle



$$\text{Area} = \frac{1}{2} \times base \times height$$
$$= \frac{1}{2} \times 10 \times 4$$
$$= 20$$

Triangle ABP we have the points $A\left(-\frac{1}{3}, 1\right)$, $B(2,1)$, $P\left(\frac{4}{9}, \frac{10}{3}\right)$

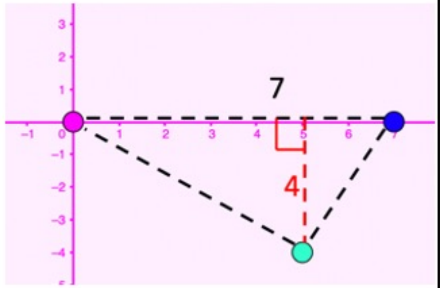
We need to form the red line to see the height of the triangle



$$\text{Area} = \frac{1}{2} \times base \times height$$
$$= \frac{1}{2}\left(2 + \frac{1}{3}\right)\left(\frac{10}{3} - 1\right)$$
$$= \frac{48}{18}$$

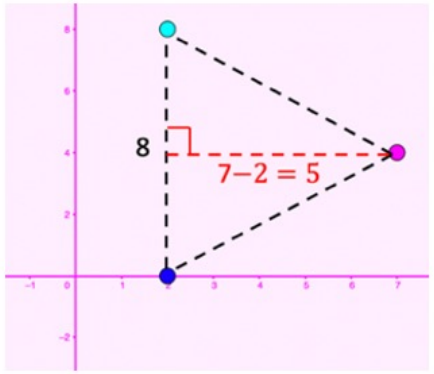
Triangle ABC formed by coordinates A(0,0), $B(7,0), C(5,-4)$

We need to form the red line to see the height of the triangle



$$\text{Area} = \frac{1}{2} \times base \times height$$
$$= \frac{1}{2} \times 7 \times 4$$
$$= 14$$

Triangle ABC formed by coordinates A (7, 4),B(2, 0), $C(2,8)$



$$\text{Area} = \frac{1}{2} \times base \times height$$
$$= \frac{1}{2}(8)(5)$$
$$= 20$$

Examples To Try

Find an equation of the line joining A (7, 4) and B (2, 0), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

i. Find the length of AB, leaving your answer in surd form

The point C has coordinates (2, t), where $t > 0$, and AC = AB.

ii. Find the value of t

iii. Find the area of triangle ABC

The straight line L has equation $3x + 2y = 17$. The point A has coordinates (0,2). The straight line M is perpendicular to L and passes through A. Line L crosses the y axis at the point B. Lines L and M intersect at the point C. Work out the area of triangle ABC

The line l_1 passes through the points P(–1, 2) and Q(11, 8)

i. Find an equation for l_1 in the form $y = mx + c$, where m and c are constants.

   The line l_2 passes through the point R(10, 0) and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

i. Calculate the coordinates of S.

ii. Hence, or otherwise, find the exact area of triangle PQR

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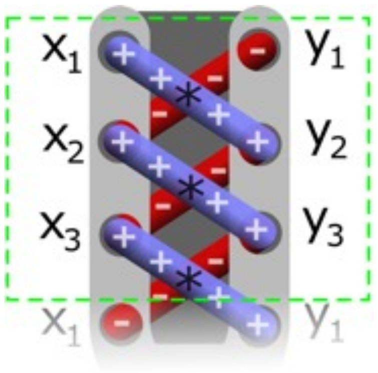
Area Hack

Did you know we can easily find the area of any n sided shape, just by knowing its coordinates? There is a VERY USEFUL formula that can find the area of ANY shape if you JUST have the coordinates. This formula is called the shoelace formula (aka shoelace algorithm or shoelace method or Gauss's area formula). It is a algorithm to determine the area of a simple polygon (a polygon that does not intersect itself and has no holes). It is called the shoelace formula because of the constant cross-multiplying for the coordinates making up the polygon, like threading shoelaces.

Imagine how cool it would be to find the area of any difficult looking shape if you have its coordinates. Well now you can ☺

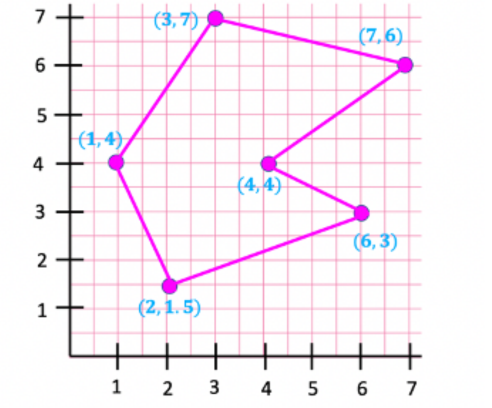
Step 1: Plots the coordinates
Step 2: Start at ANY coordinate
Step 3: Go **anti-clockwise** around the shape and write down all vertices as a vertical list. Make sure you “close the shape” at the end by re-writing the first coordinate you started with.
Step 4: Cross multiply corresponding diagonal coordinates and add. First going from left to right and then right to left



Step 5: Subtract these two answers and then divide by 2

$$\frac{\backslash - /}{2}$$

For example, we can find the area of this in around 1 minute.



Let's pick a point to start at: (2,1.5) . We go anti-clockwise.

(2,1.5)
(6,3)
(4,4)
(7,6)
(3,7)
(1,4)
(2,1.5)

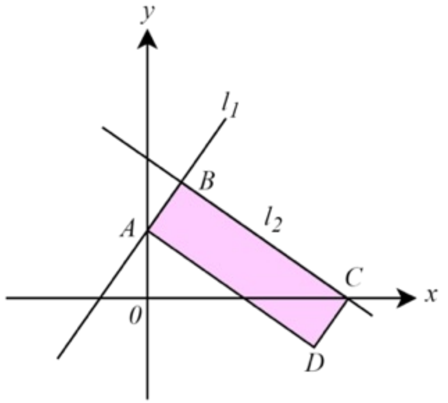
Let's colour code to explain the method.

Multiplying diagonally from left to right gives us the numbers in our first bracket	Multiplying diagonally from right to left gives us the numbers in our second bracket
(2,1.5) (6,3) (4,4) (7,6) (3,7) (1,4) (2,1.5)	(2,1.5) (6,3) (4,4) (7,6) (3,7) (1,4) (2,1.5)

$$\frac{((2\times3)+(6\times4)+(4\times6)+(7\times7)+(3\times4)+(1\times1.5)) - ((1.5\times6)+(3\times4)+(4\times7)+(6\times3)+(7\times1)+(4\times2))}{2}$$

$$= \frac{116.5 - 82}{2} = 17.25\ units^2$$

Examples To Try



The straight line l_1 has equation $2y = 3x + 7$
The line l_1 crosses the y-axis at the point A as shown

i. State the gradient of l_1
ii. Write down the coordinates of the point A.

Another straight line l_2 intersects l_1 at the point B (1, 5) and crosses the x-axis at the point C, as shown.
Given that angle ABC = 90°

i. Find an equation in the form $ax + by + c = 0$, where a, b and c are integers.
ii. The rectangle ABCD shown shaded has vertices A, B, C and D. Find the exact area of rectangle ABCD

The line l_1 has equation $4y + 3 = 2x$. The point *A* $(p, 4)$ lies on l_1

i. Find the value of the constant p

The line l_2 passes through the point C (2, 4) and is perpendicular to l_1

ii. Find an equation for l_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers

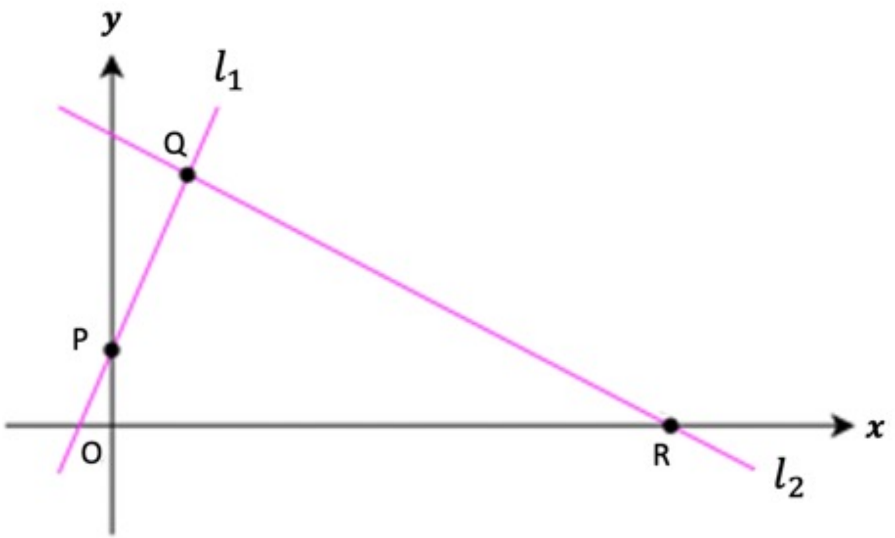
The line l_1 and the line l_2 intersect at the point D

iii. Find the coordinates of the point D
iv. Show that the length of CD is $\frac{3}{2}\sqrt{5}$

A point B lies on l_1 and the length of AB = $\sqrt{80}$
The point E lies on l_2 such that the length of the line CDE = 3 times the length of CD

v. Find the area of the quadrilateral ACBE

The points P(0, 2) and Q (3, 7) lie on the line l_1, as shown below
The line l_2 is perpendicular to l_1, passes through *Q* and crosses the *x*-axis at the point *R*, as shown.



i. Find an equation for l_2, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers,
ii. the exact coordinates of R
iii. the exact area of the quadrilateral *ORQP*, where *O* is the origin